INTERPOLATION APPROACHES TO VECTOR QUANTIZATION FOR IMAGE COMPRESSION

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Abstract — Vector Quantization (VQ) applies effectively to very low bit-rate image transmission. The high compression ratios are often balanced by limited reconstruction quality due to blockiness effects due to coarse quantization. Interpolating different codevectors can overcome such drawback by enhancing the generalization ability and the adaptiveness of the coding system at run time. Experimental results on real testbeds show the method's advantages as compared with related techniques.

I. INTRODUCTION

The growing interest in low bit-rate image transmission has recently pointed out the need for high-compression coding methods. Vector Quantization (VQ) is receiving renewed attention [1], as it can reach high compression levels. Vector Quantization represents information by placing a set of reference vectors (*prototypes*) at significant positions in the observed data space. Prototype positioning is performed according to some example-driven iterative algorithm. Some algorithms implement a probabilistic-like distribution of data over vectors [2]; other models support inter-neuron connectivity by a fixed [3] or variable [4] topology to control data partitioning. The dynamic approach described in [5] yields a uniform distribution of neurons over training data.

VQ-based coded pictures are split into many elementary blocks of fixed size (e.g., 4x4 or 8x8 pixels), representing the data samples for quantization. High compression ratios result from using a number of prototypes that is much smaller than the cardinality of the data set. VQ attains remarkable compression ratios at the cost of a reduced visual quality; tessellation brings about coarseness in reconstructed patterns. Variable-size adaptive compression can limit the tessellation drawback by pre-classifying image sub-blocks and tuning codebooks and dimensions accordingly [6-8]. A deeper generalization problem affects the family of closest-prototype schemata, due to the fact that codevector positioning is the result of a partial (often biased) sampling of the data space.

The paper describes a novel methodology that intrinsically overcomes the structural limitations of VQ by increasing representation accuracy. The proposed

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multi-best approach implements an interpolation strategy, by which multiple codevectors contribute to encoding a single pattern. In this sense, the method can be regarded as a multi-dimensional extension of neural network ensemble techniques for accurate estimation [9]. In the literature, an approach following a strategy very close to the multi-best method is presented in [10], where a multiple-prototype mechanism aims to account for multiple-cause observations, and the overall goal is the decomposition/reconstruction of event probabilities.

In order to limit the computational overhead and to ensure a satisfactory compression ratio, the paper is focused on a *two-best* encoding method, which uses a pair of codevectors for each block. A sub-optimal linear-complexity algorithm is described. The interpolation strategy is combined with a (classical) variable-size approach to attain both representation and structural adaptiveness.

As opposed to classical VQ schemata, interpolation can correctly reconstruct patterns never seen before and not included in the codebook. The generality of the overall compression method is then remarkably enhanced, as the system turns out to be less "sensitive" to the training set. In addition, the basic interpolation mechanism applies to *any* prototype-based representation schema and is independent of the vector-positioning algorithm used for training. The interpolation-based strategy shifts the problem of improving reconstruction quality from the training phase to the encoding phase, thus reducing the importance of *off-line* training in favor of *on-line* restoring ability.

The approach was applied to a significant set of natural images. Results have been evaluated from both an analytical (in terms of MSE) and a qualitative point of view. In all experiments, the method compared favorably with standard VQ approaches and with classical DCT-based techniques [11,15].

II. Vector Quantization in Image Compression

A. Basic VQ Techniques for Image Compression

In image VQ, a picture is split into M blocks; each block $\mathbf{P}_i$, $i=1,\ldots,M$ includes N pixels and defines a vector in an N-dimensional space. For example, in the case of 8-bpp gray-scale images, the space is an N-dimensional hypercube with 256 possible values for each component. Each vector $\mathbf{P}_i$ is compared with a collection of C codevectors $\mathbf{X}_j$, $j=1,\ldots,C$, defined in the same space: $\mathbf{X}_j = [x_{j1}, x_{j2},\ldots, x_{jN}]$. The codevectors represent prototypes of processed information and their ensemble forms an *alphabet* (*codebook*) of codewords. The assumption underlying data-driven training of codebooks is that a finite collection of patterns can represent consistently the actual distribution of coded data. Vector quantization compresses a sample pattern by selecting one of the available codewords following a minimum-distortion criterion. A reliable assessment criterion based on the properties of the human visual system has not yet been defined, hence Euclidean metrics will be adopted as a default distortion measure

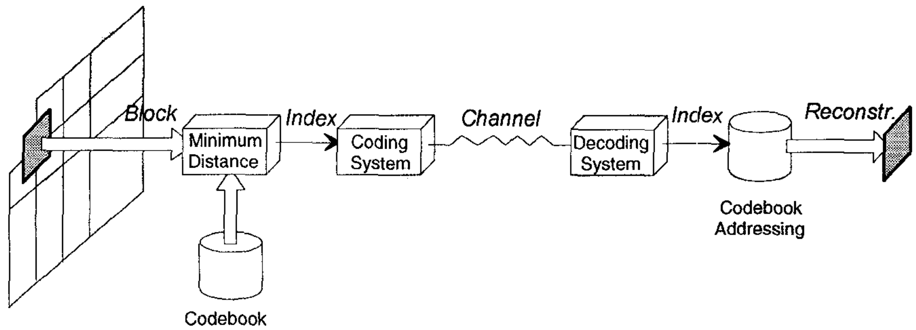

Fig. 1. VQ block diagram

$d(\mathbf{P}_i, \mathbf{X}_j)$. The codeword (Fig.1) associated with $\mathbf{P}_i$ is the vector $\mathbf{X}_k$ such that $d(\mathbf{P}_i, \mathbf{X}_k) \leq d(\mathbf{P}_i, \mathbf{X}_j)\ \forall j=1,...,C$. The compression ratio can be expressed as

$$CR_{VQ} = \frac{Nb^q}{b^c} \tag{1}$$

where b^q are the bits per pixel in the original image and b^c indicates the number of bits needed to index a codeword.

The adequacy of the alphabet plays a crucial role in VQ methods. As the final visual quality results from both the positions and the number of codewords used, the effectiveness of the training algorithm is of major importance. Suitable techniques aim to build the "best" codebook that minimizes loss in quality at a given compression ratio. Besides the *LBG* algorithm by Linde, Buso and Gray, other iterative approaches relate to neural network models and are based on *Kohonen Self-Organizing Maps* (SOMs) [3]. Within the connectionist framework, Martinetz *et al.* proposed a topology-free *Neural Gas* approach [5].

Standard VQ techniques can attain remarkable compression ratios at the cost of a rather low visual quality. In particular, it has been verified [11,12] that VQ usually results in a significant loss of details and often yields too coarse reconstructions. In order to improve performance, adaptive techniques consider the different visual significance of each region in natural images. These methods either classify patterns in advance or provide specialized alphabets for different types of regions. The best tradeoff between quality and compression is searched for punctually. Uniform regions characterized by little visual information and low frequencies allow high compression ratios; high-activity, detail-rich areas are encoded more accurately at lower compression ratios.

B. Adaptive Techniques in VQ

Classified VQ approaches process pixel blocks of the same size. A classical application of adaptive compression to VQ-based representations implies a preliminary classification of blocks. Image regions are associated with different classes; the block's internal activity (variance) is used as a classification criterion. Each class is assigned a specific codebook; codebooks can vary in both nature and size. This class of approaches can enhance quality and performance, but inflate computational complexity and demand a larger memory occupation. Moreover, the resulting quality strictly depends on both the training sets and the learning algorithm applied. The obtained compression/quality tradeoff does not always justify the considerable increase in complexity.

Variable block-size approaches to adaptivity let the dimension of input blocks vary. Images are pre-processed and split into different areas. The activity (uniformity level) is computed for each block and compared to a given threshold. If a preset uniformity level is not reached, the block is split into several sub-blocks, and the process recursively iterates on "child" blocks. A *quad tree* is progressively constructed [13]; different codebooks deal with samples having different dimensionality. Thanks to block-size adaptiveness, a quadtree-based approach considerably improves compression/quality ratio and visual quality. The eventual visual results are greatly improved, even though the corresponding increase in complexity may be significant because of block pre-processing. A more effective method will involve a tradeoff between these aspects.

III. INTERPOLATION-BASED MULTI-BEST TECHNIQUES

A. Basic Multi-Best schema

In classical VQ schemata, reconstructed patterns necessarily belong to the codevector collection, as the available alphabet(s) restrict(s) the set of possible outcomes of a VQ compression system. The multi-best methodology consists in using more than one reference vector to code an image block. An ensemble of different prototypes (*pivots*) are considered simultaneously; a reconstructed output pattern is derived from a weighted interpolation of the ensemble components. This approach allows the reconstruction of patterns that are not present in the codebook, thus considerable improvements in visual results are obtained. Moreover, the method ultimately reduces the importance of the training set and of the learning algorithm for the final quality. The ensemble of pivot vectors for the i-th pattern is called *Interpolation Set* and is defined as $W_i = \{\mathbf{W}_{i1}, \mathbf{W}_{i2}, \mathbf{W}_{iZ(i)}\}$; the ensemble cardinality, $Z(i)$, varies according to both reconstruction requirements and the pattern's characteristics. A weighting factor, w_{ij}, is associated with each vector $\mathbf{W}_{ij}$. A reconstructed pattern, $\mathbf{R}_i$, is a linear combination of the pivots considered:

$$\mathbf{R}_i = \sum_{j=1}^{Z(i)} w_{ij}\mathbf{W}_j \tag{2}$$

As results may be quite different from the basic codevectors, the system's generalization ability is significantly enhanced. A final quality cross-check between classical and multi-best VQ approaches maintains consistency and emphasizes simplicity. Multi-best will be applied whenever $d(\mathbf{P}_i, \mathbf{R}_i) \leq d(\mathbf{P}_i, \mathbf{W}_{ij})$ $\forall j=1,\ldots,C$. This relation ensures that the interpolation will provide better reconstruction results than each individual prototype. Such criterion for triggering interpolation can be refined by thresholding mechanisms without loss of generality. The method involves an increase in the amount of transmitted information, since weighting factors must be transmitted along with the indexes of the pivots used. Clearly, this affects compression performance. The compression ratio in a multi-best case can be expressed as

$$CR_{MB} = \frac{MNb^q}{\sum_{i=1}^{M}\left(\sum_{j=1}^{N_i}\left(b^c + b_j^w\right)\right)} \tag{3}$$

where: M is the number of blocks considered, b^q are the bits per pixel in the original image, b^c and b_j^w indicate the number of quantization bits for representing the codebook indexes and the weighting factor for the j-th "winner" vector, respectively. Expression (3) shows that real applications will require a tradeoff between improvement in quality and loss in compression.

Experimental evidence proves that the multi-best approach avoids coarseness and greatly limits artifacts. The basic advantage of multi-best image coding is the capability to control the quality of reconstruction without changing the codebook. Adaptive quality control can be obtained just by varying the number of pivots and the weighting procedure. Thanks to the interpolation, under a wide range of working conditions, changes in the operation environment do not require either massive retraining or additional codebook re-transmission.

In other words, the multi-best approach shifts the problem of enhancing quality from a huge VQ-training process to an adaptive coding schema. Another significant result is the reduced importance of the off-line training step. From a theoretical point of view, the interpolation mechanism makes it possible to improve quality optimization both by *a priori* information about the covered visual classes and by *a posteriori* evidence provided by each processed sample.

B. Two-best approach to Vector Quantization

A drawback of the described method lies in the possibly high loss in compression. To overcome such limitation, we consider a *two-best* solution, which encodes image sub-blocks by using two pivots for the interpolation process. The corresponding reduction in complexity allows a satisfactory increase in compression. Experimental evidence on natural images proves that a two-best approach performs satisfactorily in most cases, as more than 80% of blocks are suitably reconstructed by at most two elements ($RMSE$<10).

The two-best reconstruction process involves two phases: 1) the identification of the optimal pair of pivots (*Pivots Identification*); 2) the actual interpolation mechanism to be applied (*Interpolation Method*). Both phases can be applied in different ways, thus notably modifying the overall system's behavior. In particular, pivot selection heavily depends on the actual interpolation method applied, as a pair of codewords may be an optimal choice for one interpolation schema, but prove ineffective for another.

Therefore, interpolation will be discussed first. A reasonable approach considers the line defined by the selected pivots (*joint line*) [14]. In this case, the pattern's position can be approximated by the projection of the pattern on the line ($\mathbf{R}_j$). This *projection method* does not ensure optimality but has proved to be a good approximation in most cases. Moreover, the method involves the transmission of the scalar projection value (p_i) instead of the two weighting factors (w_{i1}, w_{i2}). The projection value has to be quantized, thus bringing about errors and distortions. Such problems can be overcome by considering the smallest projection value related to the closest pivot. This limits the saturation probability and allows higher accuracy. The *projection method* leads to a satisfactory compromise between reconstruction fidelity and compression ratio, and attains good results also in computational terms.

Let α denote the angle between $\mathbf{P}_i$ and ($\mathbf{W}_{i2}$-$\mathbf{W}_{i1}$); for practical purposes, the projection value, p_i, is normalized:

$$p_i = |\mathbf{P}_i|\cos\alpha = (\mathbf{P}_i - \mathbf{W}_{i1}) \bullet \frac{\mathbf{W}_{i2} - \mathbf{W}_{i1}}{|\mathbf{W}_{i2} - \mathbf{W}_{i1}|} \tag{4}$$

where • indicates the inner vector product. To simplify notation, we define a *Multi-Best Reconstruction* operator, *MBR*(), which unifies the method's reconstruction process analytically, as follows:

$$\mathbf{R}_i \equiv MBR(\mathbf{P}, \mathbf{W}_a, \mathbf{W}_b) = \mathbf{W}_a + p_i \frac{(\mathbf{W}_b - \mathbf{W}_a)}{|\mathbf{W}_b - \mathbf{W}_a|} \tag{5}$$

In practice, inter-pivot versors can be tabulated in advance and stored to minimize run-time computational costs. Expression (5) shows that, for each block, transmitted quantities include pivot indexes and the quantized projection value. It is worth noting that the projection may also take on negative values.

This formulation of the compression problem points out the importance of the second process phase, i.e. *Pivots Identification*. For each processed pattern, this implies the identification of the suitable couple of codevectors actually used for interpolation. Again, a smallest-distance criterion rules this selection. The identification process must locate the pair of codewords ($\mathbf{W}_a$, $\mathbf{W}_b$) in such a way that the interpolated vector $\mathbf{R}_i$ is closest to the sample vector $\mathbf{P}_i$. In principle, this might involve scanning all distinct couples of prototypes in the codebook. From a computational point of view, the exhaustive search among all possible pairs of codevectors is unrealistic; this ideal approach will be denoted by the suffix *W_W*.

A much simpler strategy can notably reduce the computational load. This technique arbitrarily picks the *best-matching* neuron (in the classical sense) as one of the two pivots. After fixing the best-matching pivot, the maximum projection algorithm is applied to identify the most suitable pair for the reconstruction. In the following, this technique will be denoted by the suffix *B_W*. Therefore, if $\mathbf{B}_i$ denotes the best-matching neuron with the pattern $\mathbf{P}_i$, the sub-optimal solution to the interpolation problem can be formally expressed as:

$$W_i^{(B\_W)} = \left(\mathbf{B}_i, \arg\left(\min_{\mathbf{W}_b \in X}\left\{\left|\mathbf{P}_i - MBR(\mathbf{P}_i, \mathbf{B}_i, \mathbf{W}_b)\right|\right\}\right)\right) \tag{6}$$

Obviously, the simplified approach yields sub-optimal results, as compared with the *W_W* exhaustive-search technique; yet the *B_W* method appears quite interesting from a practical perspective, for two main reasons. First, the mechanism represents a superset of classical VQ; the structural underlying assumption is that the closest neuron is the most likely candidate for interpolation. Second, the computational cost involved increases linearly with the number of neurons. The choice of pivoting around the closest prototype may appear arbitrary. Its validity, however, is proved by the nature of the problem itself and by statistical considerations. Experimental evidence shows that this solution offers impressive advantages over single-neuron VQ and compares favorably with the *W-W* solution, performing close to optimality in most cases.

C. Technical implementation

The technical realization of VQ-based compression also included an adaptive, variable-size approach. The adaptive approach is suggested by the fact that uniform or slowly changing patterns are suitably reconstructed also by a single-vector schema. An exhaustive application of two-best interpolation to image blocks would waste compression and computational resources. Therefore, a good approach to a quality/compression tradeoff seems to apply two-vector interpolation only to high-activity areas (i.e., high-variance blocks). This implies a size-adaptive schema, that performs a quad-tree segmentation; a block-variance threshold, σ, rules block splitting. During encoding, a picture is split into blocks of either 16x16 or 8x8 pixel size, according to internal block activity. Two-best compression applies to 8x8 blocks only. The integrated methodology combines size-adaptive with coding-adaptive mechanisms. The hybrid method nature represents a crucial features and the basis for attaining multiform adaptiveness. Fig. 2 presents an outline of the global compression schema.

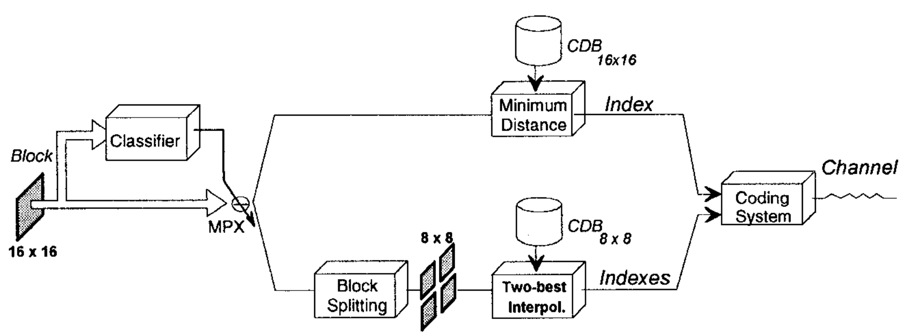

Fig.2 - The overall compression schema

IV. EXPERIMENTAL RESULTS

The experiments demonstrate the basic feature of the multi-best technique, namely an increased generalization ability due to the capability of reconstructing patterns not present in the codebook, even when they differ markedly from prototypes. The training set included three grayscale (8 bpp) 512x512 standard pictures. The test set consisted of two other pictures (*lena* and *sailboat*), specifically chosen to stress the method's generalization ability (*sailboat* has a frequency distribution quite different from that of the training samples). In all tests, we used a Self-Organizing Map with a planar topology and 256 neurons layered along a 16x16 square grid.

In the experiments using a classical VQ approach, images were split into 8x8 blocks. In multi-best experiments, instead, training pictures were split into two sets of blocks to support the size-adaptive compression schema: 16x16 blocks corresponded to uniform regions and 8x8 blocks covered regions with details. This block classification followed a variance-based criterion. In traditional VQ, as expected, more complex blocks yield less accurate reconstructions. Fig. 3 compares performances from a different perspective, plotting reconstruction quality versus compression ratio. The figure gives the results obtained by traditional VQ, by the *W_W* and the *B_W* approaches on a test image. Experiments have been performed exhaustively applying the two-best approach on all the 8x8 blocks selected by the pre-processing phase. A fixed amount of 4 bits has been adopted to encode (quantize) the block's mean and the projection's value quantization, respectively. Compression is varied by modifying the block-variance threshold σ. The graphs show that the *B_W* schema does not perform very differently from the exhaustive *W_W* schema, whereas its behavior is quite different from that of the classical approach.

The comparative analysis of the proposed methods also required qualitative evaluations of reconstructed pictures. This visual analysis confirms the performance improvement resulting from codevector interpolation. Fig.4 displays a comparison between a mono- and a multi-best reconstruction of the *lena* image (in this case, the *B_W* method was applied). The overall *low-pass effect* is strongly

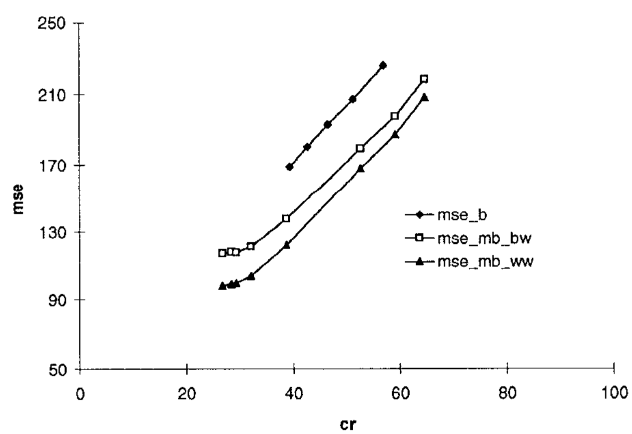

Fig.3 - Reconstruction errors vs/ compression ratios (*CR*) for the *lena* image for: *classical VQ* (*C=32-512*), *two-best B_W* and *two-best W_W* (C16=C8=256)

a) b)

Fig. 4. Reconstrcuted images of *lena* (test)
a) standard *VQ (C=256; BR=0.188 bpp, PSNR=28.26 dB)*; b) *two-best B_W* (C16=C8=256; *BR=0.197 bpp, PSNR=30.27 dB*)

reduced and the *staircase effect* was avoided, thus allowing a more natural perception of the picture. For completeness, two-best compression was also compared with traditional DCT-based [15] algorithms. Although at high bit rates JPEG attains a good quality, a dramatic loss in quality occurs at low bit rate (e.g., only 29.14 dB at BR=0.20 bpp). From a visual point of view, blockness and artifacts make JPEG useless.

V. DISCUSSION

The ultimate effect of the described intepolation method lies in enhancing the generalization ability of an image-compression system, by shifting the adaptiveness in the reconstruction process from *a-priori* to *a-posteriori* information. On the other hand, the consequent loss in compression ratio appears marginal when compared with the notable imrpovement in visual quality. In this sense the proposed method outperforms both classical VQ compression and DCT-based JPEG standards.

The overall technical set up can be further improved by including in the compression system additional adaptive methods, covering, for example, variable-size interpolation and selective coding of image blocks. The inclusion of these into an applicative environment for low bit-rate image transmission is currently being developed.

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