VLSI CIRCUITS WITH FRACTAL LAYOUT FOR SPATIAL IMAGE DECORRELATION

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ABSTRACT

The design methodology uses Peano-Hilbert 2-D curves to build up efficient circuits that destroy the spatial autocorrelation of an image. The approach applies to several crucial areas of information processing, such as cryptography and associative memories, and its validity stems from the fractal structure of the curves. The fractal curve topology drives the VLSI circuit layout, which includes many elementary cells operating at the local level. The resulting distributed architecture supports pseudo-random pixel remapping, and the self-similarity of the layout strongly simplifies electronic circuit implementations.

1. INTRODUCTION

A spatial-decorrelation process transforms a "natural" image into a noise-like 2D pattern (Fig.1) by a pseudo-random, deterministic procedure. Image decorrelation applies to crucial information-processing domains such as encryption and distributed storage of information; the need for efficient and real-time operation drives the research in electronic implementations.

This paper shows that Peano-Hilbert curves yield excellent decorrelation performance and a flexible architecture. The space-filling nature of the curves allows a modular circuit layout with inherent scalability; the chaotic behaviour resulting from the fractal layout supports the pseudo-random remapping function.

Associative storage of images is the sample application considered in this paper, without loss of generality. The "noise-like coding" model of associative memory [1] is the reference paradigm, thanks to its impressive performances in image understanding [2]. The core of the associative model's theory lies in mapping a natural image into a noise-like "key" [1]. A key is a 2D pattern whose pixels are characterized by: 1) normalization, 2) zero-mean value, and 3) mutual statistical independence. Such conditions require a pseudo-randomizing process to destroy both numerical and spatial correlations within the original image. A general solution using chaos provides numerical decorrelation [3], normalization, zero-mean value, and statistical independence at the pixel level, but it does not remove mutual correlation among neighbouring pixels.

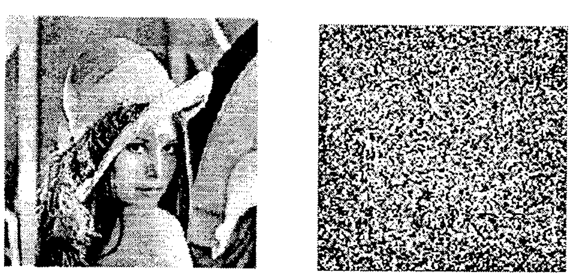

Fig.1 – Natural and spatially decorrelated images

2. FRACTAL CIRCUIT-LAYOUT IMPLEMENTATION

2.1 Fractal curves for pixel remapping

Spatial decorrelation calls for pixel remapping, which can be supported by the family of Peano-Hilbert curves [4]. The basic idea is to plot a Peano curve over the image plane: a one-dimensional rule drives the plotting, which draws the layout of the pixel-remapping circuitry. The overall decorrelation system includes many elementary units (providing numerical decorrelation), which are pairwise connected by a pixel-shifting bus according to the fractal-curve layout.

The approach exploits two theoretical features [4]: 1) a Peano curve follows a single "wire" traversing all locations independently of the actual resolution; 2) the curve can be built up by multiple reproductions of a basic elementary pattern (Fig.2).

The space-filling property ensures that the curve plot will traverse all image pixels. The self-similarity property guarantees a resolution-independent behaviour: if one divides the image into two subparts, the curve is cut at only two points. This is valid at any resolution (Fig.2). Such a feature is crucial to an efficient

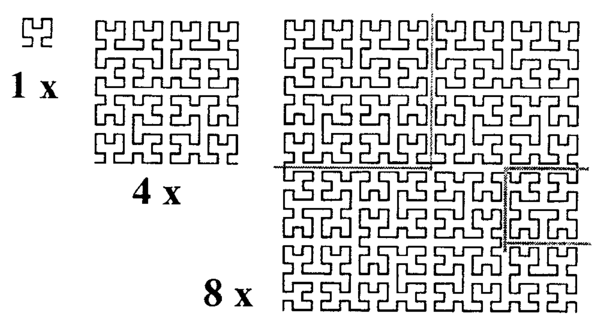

Fig.2 – Self-similarity and cutting properties of the Peano-Hilbert curves

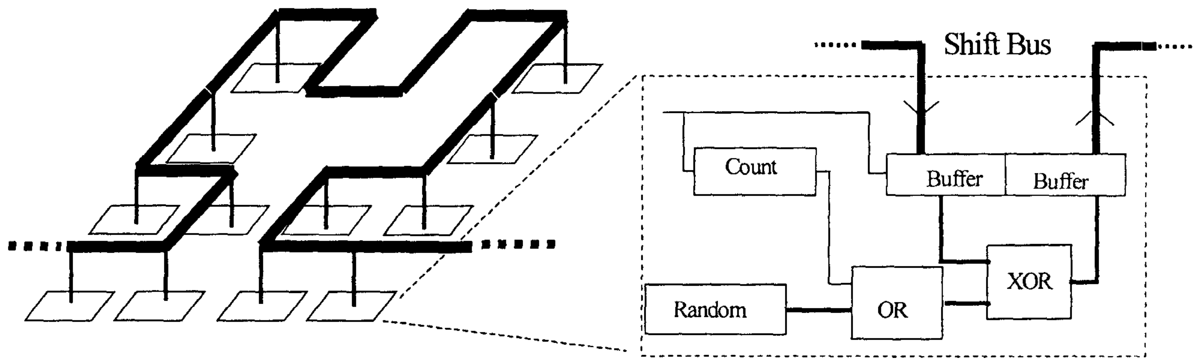

Fig.3 – The fractal layout connects pixel-processing units

implementation as it ensures that the spatial layout can be partitioned into subsections. Each curve segment communicates with the others by just two links and independently of its own size; multiple chip integration to build up very complex curves is immediate.

For the benefit of VLSI implementation, the fractal curve maps directly into a circuit layout with a cross-free, planar, one-dimensional topology. Thanks to the space-filling property, the available silicon area is fully exploited, hence the implementation is also optimal in terms of area complexity.

Each pixel-processing unit operates locally and supports the functions described in [3] by using elementary circuitry: a buffer, a counter, a pseudo-random number generator [5], and an XOR operator. Figure 3 illustrates the local-level schema of a system element. The decorrelation process iterates the following steps (δ is a shift-length parameter):

a) Cyclic-translate image pixels by δ positions according to the curve plot;

b) for each location in the layout,update its value with a pseudo-random locally computed value.

2.2 Predicting decorrelation performance

The algorithm highlights the simplicity of the system's architecture, involving two elementary operations (shift and XOR). The only parameter to be adjusted is the extension, δ, of the shifting process.

In order to evaluate the method's performance, disregarding border effects, one can consider the displacement vector, **d**, of a generic pixel in the image after applying the above algorithm. The *amplitude*, A_d, of **d** is a function of the shift extension, δ; its average value over all pixels can be computed numerically (Fig.4).

The fractal nature of the curve implies that the direction of **d** is uniformly distributed, which can be proved by considering the average phase of **d**:

$$E\{\Phi_d\}=0 \qquad (1)$$

This peculiar property of fractal curves holds at any resolution and is not shared by other remapping approaches such as linear or diagonal shift registers. The graph in Fig.4 also confirms another result that can be demonstrated, i.e., the linear relationship between displacement amplitude and shift extension (for sufficiently large values of δ). Thus the average displacement, A'_d, of a pixel is a linear function of the curve-shift amplitude, δ:

$$A'_d(\delta)=A_0+B\cdot\delta \quad (\delta > \delta_{crit}=140) \qquad (2)$$

where A_0=9 and B=0.0322.

Figure 4 and its Piece-Wise Linear approximation (1) show how many shift steps along the curve are required to attain the desired displacement of pixels.

This offers a powerful tool both for making the decorrelation method image-adaptive and, more importantly, for designing the adjustable parameter δ. The image autocorrelation function, $\Psi(a)$, evaluates the correlation between pixels lying at the distance a on the image plane:

$$\Psi(a)=\int_0^{2\pi}\int_{Image} I(x,y)I(x+a\cos\varphi, y+a\,\mathrm{sen}\,\varphi)dxdyd\varphi \qquad (3)$$

Inspection of the distribution of $\Psi(a)$ allows one to tune the decorrelation system as follows:

1) evaluate $\Psi(a)$ by standard FFTs;
2) determine from $\Psi(a)$ the highest degree, a*, of correlation that may be kept by the decorrelation process;
3) set the proper number of shift steps, δ^*, for the decorrelation system by using (1):

$$\delta^*=(a^*-A_0)/B\,.$$

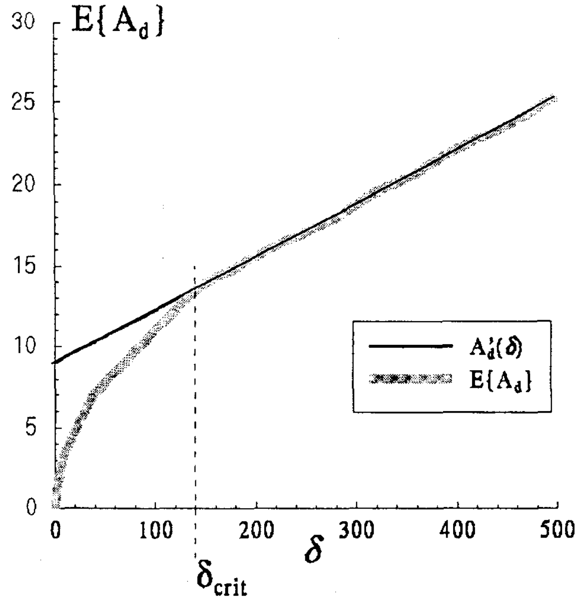

Fig.4 – Average pixel displacement as a function of curve-shift extension, δ.

3. EXPERIMENTAL RESULTS

Experiments with natural images of reasonable sizes (typ. 512x512) indicated that the average pixel correlation does not exceed a* = 24, hence a pixel-level cell requires at most a 9-bit counter (in practice, 8 bits always proved more than satisfactory). The experiments also aimed to assess the effectiveness of the image-decorrelation approach.

An assessment of the spatial randomness of a pixel distribution is in fact difficult to render quantitaively; results from the the obvious autocorrelation function may be misleading. A visual inspection of decorrelated images is often the most reliable way to validate a method's effectiveness.

In order to adopt a comparative approach, Figure 5 presents the results from three circuit approaches, each involving a pixel-pipeline structure like the one adopted in the paper. The shifting process used the same parameters (i.e., number and extension of shift steps) for all circuits; local pseudorandomizing of pixels was disabled to enable visual displacement of pixels. A linear and a diagonal shift register yielded the images shown in Fig.5 (top) and Fig.5 (center), respectively; Fig.5 (bottom) presents the result of the fractal-layout circuit approach.

The pictures indicate the better performance of the fractal methodology, which in addition exhbitis superior flexibility and modular design. Adding local pixel pseudorandomizing eventually resulted in the image shown in Fig.1b.

Fig.5 – Comparison of different image-decorrelation circuit approaches
(top: linear, center: diagonal, bottom: fractal-based)

A preliminary VLSI system implementation adopted 0.7μ AMS technology, and up to 4,096 cells could be included in a single die; the clock operating speed was ≤ 200MHz.

As final implementation steps the use of 0.35μ technology and further circuit optimization to increase the number of cells within a single chip are envisioned.

4. SUMMARY

The problem raised in this paper ultimately can be stated as follows: how can one determine whether an image is random or not? In fact, the literature does not offer an established foundation providing statistical tests about 2-D randomness tests for images.

Although such tests do exist for spatial sample distributions, such as the Kolmogorov-Smirnov 2D test, no corresponding procedure seems to exist so far for "natural" images. With respect to those methods, the image-randomness problem addresses a family of "unlabelled" signals, and does not concern classification domains.

Therefore, evaluating the quality of a (pseudo)-randomizer applied to vsual signals has to be performed visually, and this motivates the evaluation procedure adopted in the paper.

Nevertheless, current research on this subject is trying to define some objective and numerical criterion that enables one to assess a visual signal's randomness automatically.

5. REFERENCES

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