until the difference between the current boundary fields and updated boundary fields at the next iteration process reduces to the prescribed error criterion. Since this iteration process is completed within only several repetitions, the proposed method is considered to give an efficient solution for waveguide discontinuities.

**Numerical results:** For a circular aperture with radius 0.3 cm in WR-90, the performance of the proposed method is compared with that of the finite element-boundary integral method [3]. As shown in Fig. 2, the proposed method gives a result in good agreement with [3] and is more efficient in terms of computation time.

**Conclusions:** A novel iterative finite element method has been applied to a waveguide problem. The proposed method is shown to be not only simple to use but also very effective since it exploits a typical finite element procedure with only a small number of meshes and it requires only several periods of iterations. Therefore, this method is thought to represent an efficient tool for the analysis of various types of waveguide discontinuities.

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**References**


**Circuit implementation of K-winner machine**

S. Ridella, S. Rovetta and R. Zunino

The K-winner machine (KWM) model for supervised classification enhances vector quantisation by characterising classification outcomes with confidence levels. Each data-space location is assigned a specific local bound to the error probability. Structural simplicity makes the implementation compatible with circuitry for classical VQ, and features high speed and efficiency.

**KWM model:** A prototype-based schema spans the data space by a set of reference positions (‘prototypes’, ‘codewords’). The maximum similarity drives the categorisation process, which classifies each sample with the class of the best-matching prototype. Thus vector quantisation (VQ) involves a winner-takes-all (WTA) schema, and partitions the data space into as many Voronoi sub-regions as the number of prototypes. The samples lying in a region are classified according to the related prototype. WTA-based categorisation is not usually characterised by a confidence measure: as far as classification points concerned, all points within a region are equivalent and with an equal confidence value.

The K-winner machine (KWM) overcomes such a drawback by taking into account, for each test sample, a larger set of prototypes including K elements (K > 1). Similarly to WTA, the KWM uses the ‘winning’ prototype to set classification; however, it also seeks the largest number K of best-matching prototypes that agree with the winner. The level of agreement depends on the test location, hence each data point yields a specific value of K. The basic assumption is that a point with a large K value denotes high confidence in the associate classification. The KWM embeds classical WTA in the minimal case K = 1: when even the second-best guess disagrees with the winning candidate, confidence reaches its minimum. A WTA classifier and a KWM involve the same representation structure, i.e. a set of class-calibrated prototypes positioned in the data space by some VQ algorithm. Thus KWM training does not differ from any conventional VQ-classifier setup. Instead, the KWM run-time operation for classifying a sample x can be outlined as follows:

(i) compute the distance d_p(x) between x and each prototype w_p, p = 1, ..., N_p;
(ii) sort the list of prototypes in order of increasing d_p;
(iii) work out the largest K value such that Class(w_p) = Class(w_q) for all q = 1, ..., K;
(iv) a classify x according to the winning prototype, C(x) = Class(w_p) (WTA classification).

b prompt K as the confidence level for the present classification outcome.

Measuring the agreement among prototypes (step (iii)) helps predict the generalisation performance: it can be proved [1] that the Vapnik-Chervonenkis dimension [2] of a K-winner machine can be computed exactly as

\[ d_{VC} = \left[ \frac{N_p}{K} \right] \]

(1)

Therefore, by using eqn. 1 and basic results from generalisation theory [1], we set a bound on the KWM classifier’s error probability \( \eta \), given by

\[ \eta = \nu + \sqrt{\frac{1}{N_p} \left[ \frac{d_{VC}}{1 + \ln \left( \frac{2N_p}{d_{VC}} - \ln \frac{\eta}{\delta} \right)} \right]} \]

(2)

where \( \nu \) is the classification error for a training set including \( N_p \) samples, and \( \eta \) (typ. = 0.05) is a confidence factor.

**Fig. 1 KWM results on Gaussian-mixture testbed**

a Training data (three classes)
b Confidence map; darker areas denote higher K and higher confidence; bright regions (low K) span class boundaries

In summary, for each test location in the data space, we first compute the local K value, then by using [1, 2] we determine a bounded estimate of the expected error probability. The opportunity to predict the generalisation ability analytically and at the local level represents a crucial advantage of the KWM model. A KWM differs from a voting schema substantially, as prototypes
are not trained independently of one another, and the number $K$ depends on each specific sample. Incidentally, when $K$ increases, i.e. when more neighbours agree on a sample classification, the associate error probability (eqn. 2) decreases, thus confirming the intuitive expectation that a larger consensus yields higher confidence. The operation on a Gaussian-mixture testbed (Fig. 1) shows that the 'confidence map' spanned by $K$ renders and supports the class distribution consistently.

The actual KWM circuitry is illustrated in Fig. 3. After reset, the first clock cycle yields the winning prototype and classifies the sample according to CL(1) (step (i)). The D flip-flop FFI triggers FF2 to store Class(w). Further clock cycles yield the successive prototypes in the sorted list from the sorting chip. These are progressively counted until the XOR circuitry detects a difference between CL(w) (i.e. the class of the 4th prototype) and the contents of FF2 (the class of the winner). Then the stop signal becomes low, and the status of count gives the final value of $K$.

The current implementation of the KWM circuit includes 200 prototypes, hence an 8 bit counter is required. The basic devices have already been fabricated using 1 μm CMOS technology [3, 4], whereas the elementary KWM circuitry has been successfully simulated at layout level using HSPICE level 13. Current research is being carried out into integrating the three subsystems within a single VLSI chip.

**Conclusions:** The KWM model greatly enhances VQ classifiers by providing a measure of the confidence in the categorisation result. Consequently, the electronic implementation is straightforward and takes advantage of basic VQ-support building blocks. The importance of the final application domain (handwritten OCR) justifies the implementation effort.

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**Neural network approach to graph colouring**

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A neural network for colouring a graph of $N$ nodes is proposed which uses only $N$ neurons and $N^2$ weights. In contrast, $N^2$ neurons and $N^2$ interconnections are required by Hopfield net based approaches. Experiments with a breadboard realisation using discrete components yielded promising results.

**Introduction:** Neural networks have evolved in the last two decades into a new paradigm for solving difficult optimisation problems. The literature is replete with neural network solutions for combinatorial optimisation tasks, most of which are based on the Hopfield net [1]. One such task is the graph colouring problem (GCP), which arises in several contexts, including assignment, resource allocation, and job scheduling. The GCP involves assigning a colour to each node in a graph $G$ with $N$ nodes, such that no two adjacent nodes have identical colours. While four colours are sufficient to colour a planar graph, $N$ colours would be needed in the case of a clique of $N$ nodes. The colour assigned to a node can thus be represented by an integer in the range 1 - $N$.

The GCP for any graph with $N$ nodes can be mapped onto a Hopfield net with $N \times N$ neurons. Let $Y_i$ denote the output of the neuron in row $i$ and column $j$ where $i$ and $j$ range from 1 to $N$. $Y_j$ is either 0 or 1; if $Y_i = 1$, then this indicates that node $i$ in the graph is assigned colour $j$. Therefore, in a valid colouring, only one $Y_i$ in each row is equal to 1.