Verification of Graph Transformation Systems with Whole Neighbourhood Operations

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Motivation

Overall Goal:

Provide an high-level automatic decision procedure for the correctness of systems, e.g. protocols.

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- Model states and behaviour of the system by graphs and graph transformation rules.
- Specify sets of erroneous states.
- Automatically check if an erroneous state is reachable.

  $\rightsquigarrow$ Undecidable in general.

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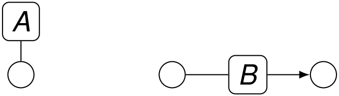
This talk is about extending the framework with an alternative rewriting formalism (accepted at RP 2014).

Hypergraphs

Definition (Hypergraph)

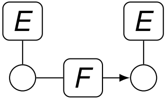
A Λ-*hypergraph* consists of a set of nodes V, a set of edges E, a connection function $c : E \to V^*$ and a labelling function $l : E \to \Lambda$. The length of the node sequence is determined by the label.

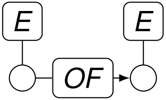


Example - Dining Philosphers

Dining Philosphers, similar to [Namjoshi, Tefler]:

- A philosopher can be hungry (*H*), eating (*E*) or thinking (*T*).
- Network structure is arbitrary.
- Forks can be free (*F*) or owned (*OF*).
- To eat a philosopher has to own all forks "within reach".
- Can a situation be reached, where two philosophers sharing a fork eat at the same time?





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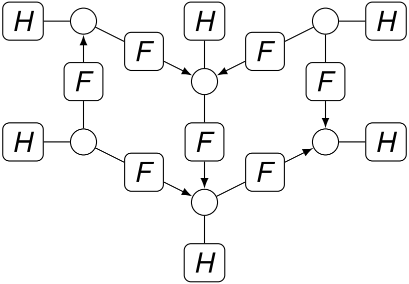

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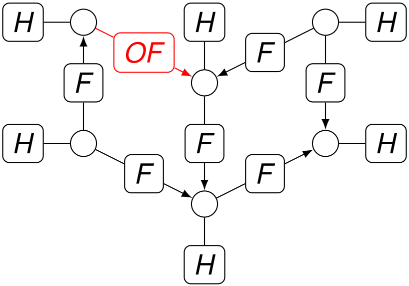

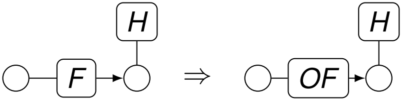

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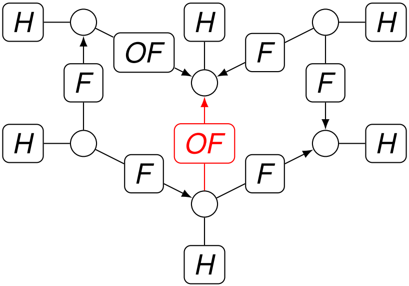

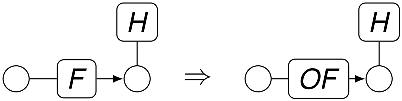

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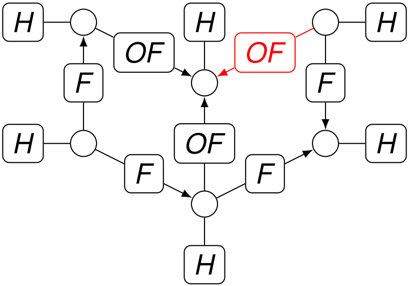

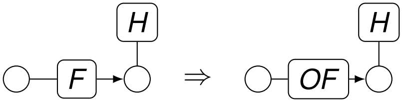

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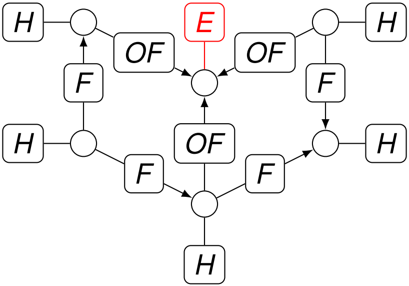
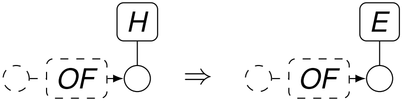
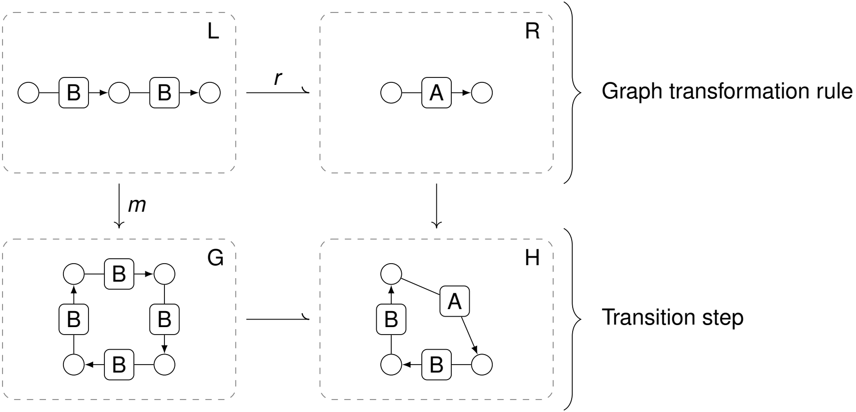




Hypergraph Rewriting

Single Pushout Approach

As rewriting formalism we use SPO with partial morphisms $r : L \rightharpoonup R$ as rules and total, injective matches.



Universal Quantification

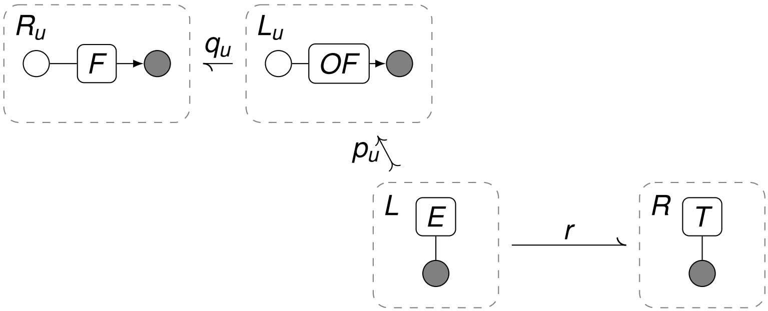
Definition (Universally quantified rules)

A universally quantified rule is a pair (r, U), where $r : L \rightharpoonup R$ is a partial morphism and U is a finite set of universal quantifications.
A universal quantification is a pair $(p_u, q_u) \in U$ where:

- $p_u : L \rightarrowtail L_u$ is a total injective morphism
- $q_u : L_u \rightharpoonup R_u$ is a partial morphism which is injective on $p_u(L)$

$$
\begin{tikzcd}
R_u & L_u \arrow[l, harpoon] & & \\
\vdots & \vdots & L \arrow[ul, tail] \arrow[dl, tail] \arrow[r, harpoon] & R \\
R_v & L_v \arrow[l, harpoon] & &
\end{tikzcd}
$$

Instantiation - Example



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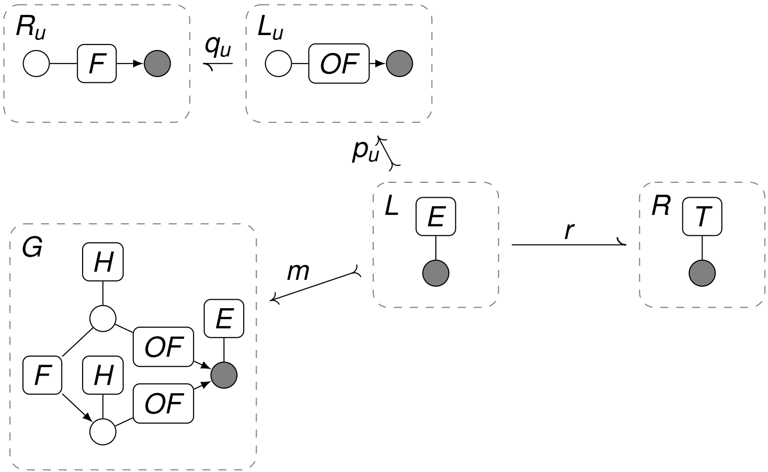

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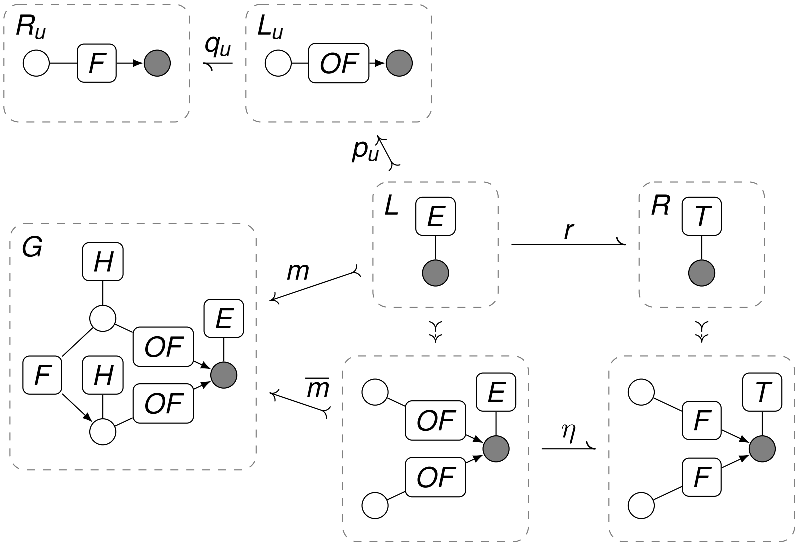

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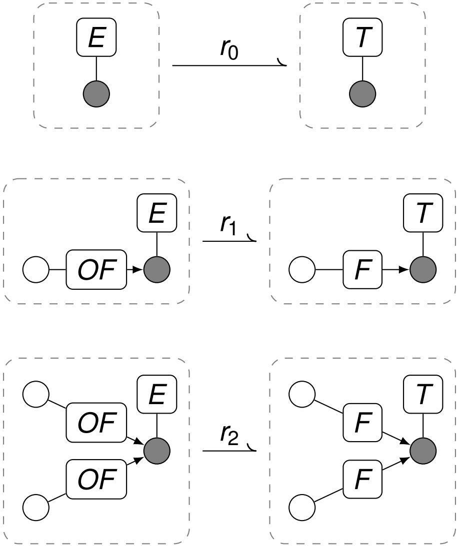

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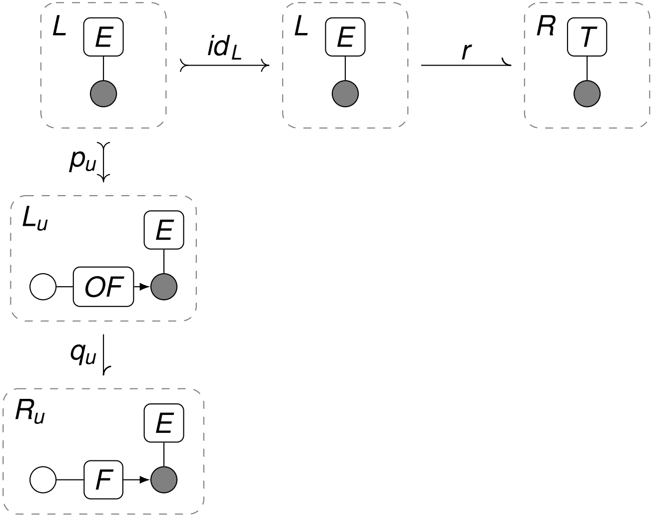

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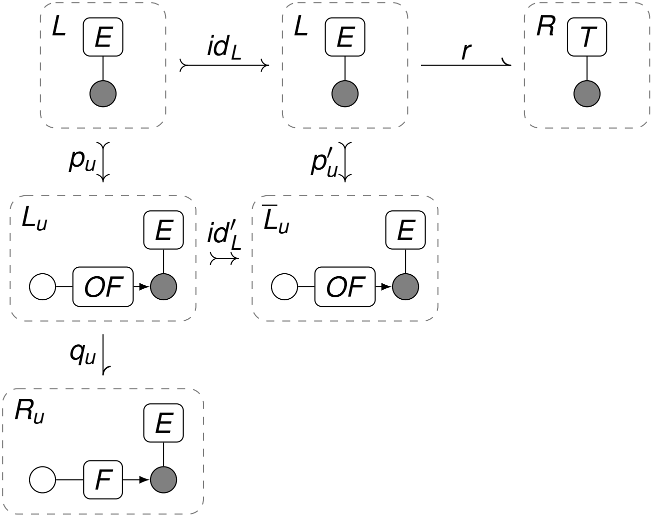

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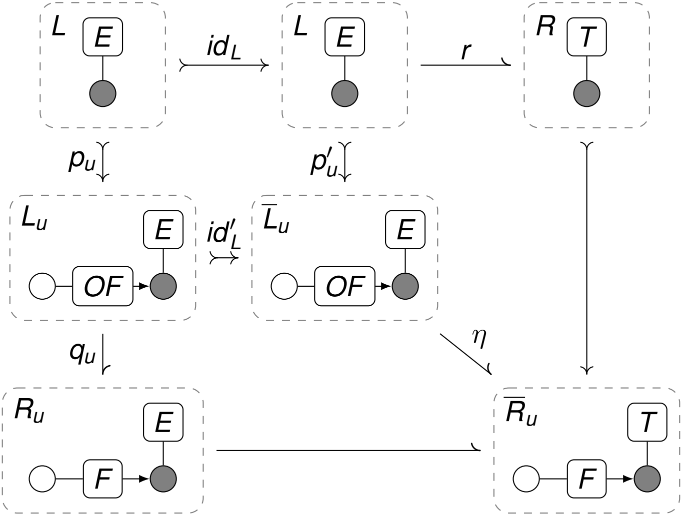


Rule Application

Definition (Rule application)

Let ρ be a universally quantified rule. We say that ρ is applicable to a graph G, if there is an instantiation η and a match m, such that the neighbourhood of every quantified node is matched.

$$
\begin{tikzcd}
\overline{L} \arrow[r, "\eta"] \arrow[d, tail, "m"'] & \overline{R} \arrow[d] \\
G \arrow[r] & H
\end{tikzcd}
$$

Analysis

How to analyse such systems, while retaining an infinite state space?

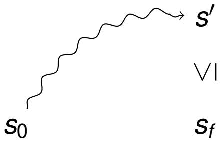
$\rightsquigarrow$ We use the theory of well-structured transition systems [CONCUR 2014], for which coverability is decidable [Finkel, Schnoebelen], [Abdulla et al.].

Coverability Problem

Let $\mathcal{T} = (S, \Rightarrow)$ be a transition system, s_0 the initial state and $\leq$ an order on states.

Coverability Problem

Given s_f, is there a state s' such that $s_0 \Rightarrow^* s'$ and $s_f \leq s'$?

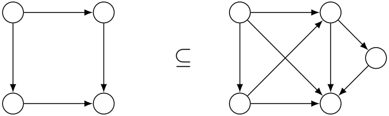


s_f is coverable

Subgraphs for Coverability

Definition (Subgraph)

A graph G is a subgraph of a graph G', if G can be obtained from G' by a sequence of node deletions and edge deletions.



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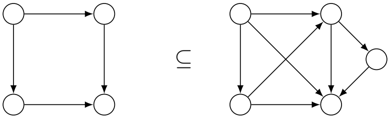

Initial Problem:

Is one of the following graphs coverable?

Condition 1: Well-quasi-order

Definition (Well-quasi-order)

A reflexive, transitive relation $\leq$ is a well-quasi-order if:

- In every infinite sequence $x_0, x_1, x_2, x_3, \ldots$ there exist indices $i < j$ such that $x_i \leq x_j$.

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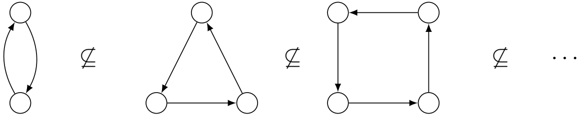
The subgraph ordering is

- not a well-quasi-order on all graphs.



- a well-quasi-order on the set of graphs $\mathcal{G}_k$ with bounded longest (undirected) paths [Ding][Meyer].

Condition 2: Monotonicity

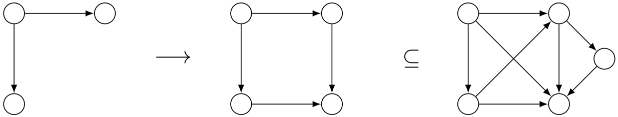
Definition (Monotonicity)

A graph transformation system is monotone if:
whenever $G_1 \subseteq H_1$ and $G_1 \Rightarrow G_2$, there exists a H_2 such that $H_1 \Rightarrow^* H_2$ and $H_2 \subseteq G_2$.

$$
\begin{array}{ccc}
H_1 & \stackrel{*}{\Longrightarrow} & H_2 \\
\cup\!| & & \cup\!| \\
G_1 & \Longrightarrow & G_2
\end{array}
$$

Monotonicity and Subgraphs

Every GTS without negative application conditions is naturally a *Q*-restricted WSTS.



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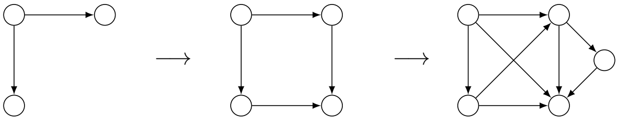

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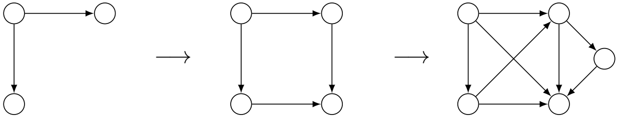


However, universally quantified rules may violate the compatibility condition.

$\rightsquigarrow$ Leads to over-approximation.

Solving Coverability by a Backward Search

We use the well-known backward search for WSTS:

- It computes the set of minimal representatives $\mathcal{W}$ of all graphs that can cover an error.
- If there is no $G' \in \mathcal{W}$ with $G' \subseteq G$, then no error graph is coverable (within $\Rightarrow_{\mathcal{G}_k}$) from G.
- We can use $\Rightarrow$ instead of $\Rightarrow_{\mathcal{G}_k}$, but lose the guarantee of termination.

Backward steps and Quantification

In [CONCUR 2014] we showed how a backward step can be computed.
However, a rule can have infinitely many instantiations!

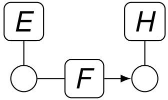
Proposition

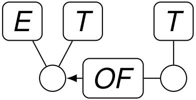
In each backward step, only finitely many instantiations have to be applied backwards.

$\rightsquigarrow$ For each rule and graph there is a constant, such that results for instantiations of at least that size are subsumed by results for smaller instantiations.

Implementation

We integrated the presented formalism in the Tool UNCOVER.
It takes ~1 second to compute 12 minimal errors for the Dining Philosophers example.





Ongoing and Future Work

Related work:

- Adaptive star grammars. [Drewes at al.]
  - $\rightsquigarrow$ Our formalisms can be seen as an extension.
- Technical similarities with amalgamation.
  [Boehm, Fonio, Habel]

Future work:

- Is our over-approximation precise enough? How can we increase precision?
- Can this approach be transferred to other orders, e.g. minors?
- With which order are universally quantified rules monotone?
- Use graph patterns to represent set of graphs?
  cf. [Saksena, Wibling, Jonsson]

Thank you for your attention!

Any Questions?