LW - Grammatica CF

Prog ::= Decs **main**() Stats.
Decs ::= Dec | Dec Decs.
Dec ::= VDec | FDec.
VDec ::= PDec | Type Id = Exp;
PDec ::= Type Id;
FDec ::= Type Id(PDecs) {[**with** Decs **do**] Stats **result** Exp} |
void Id(PDecs) {[**with** Decs **do**] Stats}
PDecs ::= PDec | PDec PDecs.
Type ::= **int** | **bool**.
Stats ::= Stat | Stat Stats.
Stat ::= Exp; | **while** (Exp) {Stats} | **if** (Exp) {Stats} **else** {Stats}|
?Id | **!**Exp.
Exp ::= Id | Id = Exp | Id(Exps) | Exp + Exp | …
Exps ::= Exp | Exp Exps.

Funzioni di Semantica Statica

$$
\begin{array}{llll}
\llbracket \_ \rrbracket^{s}_{Prog} & : L_{Prog}(LW) & \to_P \{*\} & \\
\llbracket \_ \rrbracket^{s}_{Decs} & : L_{Decs}(LW) \times Env^s \times Env^s & \to_P Env^s & \\
\llbracket \_ \rrbracket^{s}_{Dec} & : L_{Dec}(LW) \times Env^s \times Env^s & \to_P Env^s & \llbracket \_ \rrbracket^{s}_{Dec}(d,\rho,\rho_l) = \llbracket \_ \rrbracket^{s}_{Decs}(d,\rho,\rho_l) \\
\llbracket \_ \rrbracket^{s}_{VDec} & : L_{VDec}(LW) \times Env^s \times Env^s & \to_P Env^s & \llbracket \_ \rrbracket^{s}_{VDec}(d,\rho,\rho_l) = \llbracket \_ \rrbracket^{s}_{Decs}(d,\rho,\rho_l) \\
\llbracket \_ \rrbracket^{s}_{PDecs} & : L_{PDecs}(LW) \times Env^s \times Env^s & \to_P Env^s & \llbracket \_ \rrbracket^{s}_{PDecs}(d,\rho,\rho_l) = \llbracket \_ \rrbracket^{s}_{Decs}(d,\rho,\rho_l) \\
\llbracket \_ \rrbracket^{s}_{PDec} & : L_{PDec}(LW) \times Env^s \times Env^s & \to_P Env^s & \llbracket \_ \rrbracket^{s}_{PDec}(d,\rho,\rho_l) = \llbracket \_ \rrbracket^{s}_{Decs}(d,\rho,\rho_l) \\
\llbracket \_ \rrbracket^{s}_{FDec} & : L_{FDec}(LW) \times Env^s \times Env^s & \to_P Env^s & \llbracket \_ \rrbracket^{s}_{FDec}(d,\rho,\rho_l) = \llbracket \_ \rrbracket^{s}_{Decs}(d,\rho,\rho_l) \\
\llbracket \_ \rrbracket^{s}_{Stats} & : L_{Stats}(LW) \times Env^s & \to_P \{*\} & \\
\llbracket \_ \rrbracket^{s}_{Stat} & : L_{Stat}(LW) \times Env^s & \to_P \{*\} & \llbracket \_ \rrbracket^{s}_{Stat}(st,\rho) = \llbracket \_ \rrbracket^{s}_{Stats}(st,\rho) \\
\llbracket \_ \rrbracket^{s}_{Exp} & : L_{Exp}(LW) \times Env^s & \to_P Types & \llbracket \_ \rrbracket^{s}_{Exp}(e,\rho) = \llbracket \_ \rrbracket^{s}_{Exps}(e,\rho) \\
\llbracket \_ \rrbracket^{s}_{Exps} & : L_{Exps}(LW) \times Env^s & \to_P Types^+ &
\end{array}
$$

$Env^s = [L_{Id}(LW) \to_P Types]_{Fin}$, dove $Types = BTypes \cup (BTypes^+ \times RTypes)$, dove $BTypes = \{int, bool\}$ e $RTypes = BTypes \cup \{void\}$

Notazione per funzioni parziali a dominio finito $f \in [X \to_P A]_{Fin}$

se f è $[]$, $Dom(f) = \emptyset$

se f è $[a_1/x_1,\ldots, a_n/x_n]$ con tutte le x_i *distinte*, $Dom(f) = \{x_1,\ldots,x_n\}$ e $f(x_i)=(a_i)$

se f è $g[a/\underline{x}]$, $Dom(f) = Dom(g) \cup \{\underline{x}\}$ e $f(x)= g(x)$ se $x \neq \underline{x}$, altrimenti $f(\underline{x})=a$

se f è $g[h]$, $Dom(f) = Dom(g) \cup Dom(h)$ e $f(x)= h(x)$ se $x \in Dom(h)$, altrimenti $f(x)= g(x)$

se f è $g \backslash \underline{X}$, $Dom(f) = Dom(g) \backslash \underline{X}$ e $f(x)= g(x)$

Regole della Sem. Statica per Decs

$$\frac{\llbracket d \rrbracket^s(\rho,\ \rho_l) = \rho_{loc} \qquad \llbracket ds \rrbracket^s(\rho,\ \rho_{loc}) = \rho_0}{\llbracket d\ \ ds \rrbracket^s(\rho,\ \rho_l) = \rho_0}$$

$$\frac{\llbracket e \rrbracket^s(\rho[\rho_l])=t}{\llbracket t\ id = e; \rrbracket^s(\rho,\rho_l) = \rho_l[t/id]}\ id \notin Dom(\rho_l) \qquad \frac{}{\llbracket t\ \ id; \rrbracket^s(\rho,\rho_l) = \rho_l[t/id]}\ id \notin Dom(\rho_l)$$

$$\frac{\llbracket pl \rrbracket(\rho[\rho_l],[(lt,void)/f]) = \rho_{loc} \qquad \llbracket sts \rrbracket(\rho[\rho_l]\ [\rho_{loc}]) = *}{\llbracket \mathbf{void}\ \ f(pl)\ \{sts\} \rrbracket(\rho,\rho_l) = \rho_l[(lt,void)/f]} \quad \begin{array}{c} lt=types(pl) \\ f \notin Dom(\rho_l) \end{array}$$

$$\frac{\llbracket pl \rrbracket(\rho[\rho_l],[(lt,t)/f]) = \rho_{loc} \quad \llbracket sts \rrbracket(\rho[\rho_l]\ [\rho_{loc}]) = * \quad \llbracket e \rrbracket(\rho[\rho_l]\ [\rho_{loc}]) = t}{\llbracket t\ \ f(pl)\ \{sts\ \mathbf{result}\ e\} \rrbracket(\rho,\rho_l) = \rho_l[(lt,t)/f]} \quad \begin{array}{r} lt=types(pl) \\ f \notin Dom(\rho_l) \\ t \neq \mathbf{void} \end{array}$$

$$\frac{\llbracket pl \rrbracket(\rho[\rho_l],[(lt,void)/f]) = \rho_{loc}\ \llbracket ds \rrbracket(\rho[\rho_l],\ \rho_{loc}) = \rho'_{loc}\ \llbracket sts \rrbracket(\rho'') = *}{\llbracket \mathbf{void}\ \ f(pl)\ \{\mathbf{with}\ ds\ \mathbf{do}\ sts\} \rrbracket(\rho,\rho_l) = \rho_l[(lt,void)/f]} \quad \begin{array}{r} lt=types(pl) \\ f \notin Dom(\rho_l) \\ \rho'' = \rho[\rho_l]\ [\rho'_{loc}] \end{array}$$

$$\frac{\llbracket pl \rrbracket(\rho[\rho_l],[(lt,t)/f]) = \rho_{loc}\ \llbracket ds \rrbracket(\rho[\rho_l],\ \rho_{loc}) = \rho'_{loc}\ \llbracket sts \rrbracket(\rho'') = *\ \ \llbracket e \rrbracket(\rho'') = t}{\llbracket t\ \ f(pl)\ \{\mathbf{with}\ ds\ \mathbf{do}\ sts\ \mathbf{result}\ e\} \rrbracket(\rho,\rho_l) = \rho_l[(lt,t)/f]} \quad \begin{array}{r} lt=types(pl) \\ f \notin Dom(\rho_l) \\ \rho'' = \rho[\rho_l]\ [\rho'_{loc}] \\ t \neq \mathbf{void} \end{array}$$

Regole della Sem. Statica per Exps, Stats e Prog

$$\frac{[\![e]\!]^s(\rho) = t \qquad [\![es]\!]^s(\rho) = lt}{[\![e\ \ es]\!]^s(\rho) = t\ lt}$$

$$\frac{}{[\![id]\!]^s(\rho) = \rho(id)}\ \begin{array}{c} id \in Dom(\rho) \\ \rho(id) \in BTypes \end{array}$$

$$\frac{[\![e]\!]^s(\rho) = t}{[\![id = e]\!]^s(\rho) = t}\ \rho(id) = t$$

$$\frac{[\![es]\!](\rho) = lt}{[\![id(es)]\!](\rho) = t}\ \rho(id) = (lt,t)$$

$$\frac{[\![e_1]\!](\rho) = int \quad [\![e_2]\!](\rho) = int}{[\![e_1 + e_2]\!](\rho) = int}$$

etc...

$$\frac{[\![e]\!]^s(\rho) = t}{[\![e;]\!]^s(\rho) = *}$$

$$\frac{[\![e]\!]^s(\rho) = bool \qquad [\![sts]\!]^s(\rho) = *}{[\![\textbf{while}\ (e)\ \{sts\}\ ]\!]^s(\rho) = *}$$

$$\frac{[\![e]\!]^s(\rho) = t}{[\![\textbf{!}e]\!]^s(\rho) = *}$$

$$\frac{[\![e]\!]^s(\rho) = bool \qquad [\![sts_1]\!]^s(\rho) = * \quad [\![sts_2]\!]^s(\rho) = *}{[\![\textbf{if}\ (e)\ \{sts_1\}\ \textbf{else}\ \{sts_2\}\ ]\!]^s(\rho) = *}$$

$$\frac{}{[\![?id]\!]^s(\rho) = *}\ \rho(id) \in BTypes$$

$$\frac{[\![ds]\!]([],[]) = \rho \qquad [\![sts]\!]^s(\rho) = *}{[\![ds\ \textbf{main()}\ sts]\!]^s(\rho) = *}$$

Funzioni di Semantica Denotazionale

$[\![\_]\!]_{Prog} : L_{Prog}(LW) \times Input \to_P Output$

$[\![\_]\!]_{Decs} : L_{Decs}(LW) \times Env \times States \to_P Env \times States$

$[\![\_]\!]_{Dec} : L_{Dec}(LW) \times Env \times States \to_P Env \times States$ $\qquad [\![\_]\!]_{Dec}(d,r,s) = [\![\_]\!]_{Decs}(d,r,s)$

$[\![\_]\!]_{VDec} : L_{VDec}(LW) \times Env \times States \to_P Env \times States$ $\qquad [\![\_]\!]_{VDec}(d,r,s) = [\![\_]\!]_{Decs}(d,r,s)$

$[\![\_]\!]_{PDecs} : L_{PDecs}(LW) \times Env \times States \to_P Env \times States$ $\qquad [\![\_]\!]_{PDecs}(d,r,s) = [\![\_]\!]_{Decs}(d,r,s)$

$[\![\_]\!]_{PDec} : L_{PDec}(LW) \times Env \times States \to_P Env \times States$ $\qquad [\![\_]\!]_{PDec}(d,r,s) = [\![\_]\!]_{Decs}(d,r,s)$

$[\![\_]\!]_{FDec} : L_{FDec}(LW) \times Env \times States \to_P Env \times States$ $\qquad [\![\_]\!]_{FDec}(d,r,s) = [\![\_]\!]_{Decs}(d,r,s)$

$[\![\_]\!]_{Type} : L_{Type}(LW) \to Set$

$[\![\_]\!]_{Stats} : L_{Stats}(LW) \times Env \times States \to_P States$

$[\![\_]\!]_{Stat} : L_{Stat}(LW) \times Env \times States \to_P States$ $\qquad [\![\_]\!]_{Stat}(st,r,s) = [\![\_]\!]_{Stats}(st,r,s)$

$[\![\_]\!]_{Exp} : L_{Exp}(LW) \times Env \times States \to_P Value \times States$ $\qquad [\![\_]\!]_{Exp}(e,r,s) = [\![\_]\!]_{Exps}(e,r,s)$

$[\![\_]\!]_{Exps} : L_{Exps}(LW) \times Env \times States \to_P Value^{+} \times States$

$Env = [L_{Id}(LW) \to_p Fun \cup Loc\,]_{Fin}$, dove

$Loc = \cup_{T \in BTypes}\, Loc_T$

$Fun = [Arg \to_p Res]$, $Arg = [\mathbb{Z} | \mathbb{B}]^{+} \times States$ e $Res = [\mathbb{Z} | \mathbb{B} | \{*\}] \times States$

$States = Input \times Output \times [Loc \to_p Value]_{Fin}$, dove

$Value = \cup_{T \in BTypes} [\![T]\!]$ e $[\![\mathbf{int}]\!] = \mathbb{Z}$, $[\![\mathbf{bool}]\!] = \mathbb{B}$, $[\![\mathbf{void}]\!] = \{*\}$

$Input = Output = Value^{*}$

$Nuova \subseteq Loc \times Env \times States$ definito da $Nuova(l,r,s)$ sse $l \notin (Im(r) \cup Dom(s))$

Regole della Sem. Denot. per Decs 1

$$\frac{}{[\![\textbf{void}\ f(T\ x)\ \{[\textbf{with}\ ds\ \textbf{do}]sts\}]\!](r,s) = (r[F/f],s)}$$

F è definita induttivamente da tutte le regole della semantica più le seguenti **2** A [A_w se c'è il with] e B, per leggibilità riportate nel riquadro

$$A\ \frac{[\![sts]\!]\,(r[\perp/f,l_x/x],s_c[v/l_x]\,) = s'}{F(v,s_c)\ = (*,s'\backslash\{l_x\})}\ Nuova(l_x,r,s_c) \qquad B\ \frac{[\![e]\!](r_0[\perp/f],s_0) = (u\ ,s') \qquad F(u,s')\ = (*,s'')}{[\![f(e)]\!]\,(r_0[\perp/f],s_0) = (*,s'')}$$

$$A_w\ \frac{[\![ds]\!](r[\perp/f,l_x/x],s_c[v/l_x]\,) = r_1,s_1 \quad [\![sts]\!]\,(r_1,s_1) = s'}{F(v,s_c)\ = (*,s'\backslash\{l_x\})}\ Nuova(l_x,r,s_c)$$

$$\frac{}{[\![RT\ f(T\ x)\ \{[\textbf{with}\ ds\ \textbf{do}]sts\ return\ e\}]\!](r,s) = (r[F/f],s)}$$

F è definita induttivamente da tutte le regole della semantica più le seguenti **2** A [A_w se c'è il with] e B, per leggibilità riportate nel riquadro

$$A\ \frac{[\![sts]\!]\,(r[\perp/f,l_x/x],s_c[v/l_x]\,) = s_1 \quad [\![e]\!](r\,[\perp/f,l_x/x],s_1) = (u\ ,s')}{F(v,s_c)\ = (u,s'\backslash\{l_x\})}\ Nuova(l_x,r,s_c)$$

$$A_w\ \frac{[\![ds]\!](r[\perp/f,l_x/x],s_c[v/l_x]\,) = r_1,s_1 \quad [\![sts]\!]\,(r_1,s_1) = s_2 \quad [\![e]\!](r_1,s_2) = (u\ ,s')}{F(v,s_c)\ = (u,s'\backslash\{l_x\})}\ Nuova(l_x,r,s_c)$$

$$B\ \frac{[\![e]\!](r_0[\perp/f],s_0) = (v\ ,s') \qquad F(v,s')\ = (u,s'')}{[\![f(e)]\!]\,(r_0[\perp/f],s_0) = (u,s'')}$$

Regole della Sem. Denot. per Decs ed Exps

$$\frac{[\![d]\!](r,s) = (r',s') \qquad [\![ds]\!](r',s') = (r'',s'')}{[\![d\ \ ds]\!](r,s) = (r'',s'')}$$

$$\frac{}{[\![t\ x]\!](r,s) = (r[l/x],s[\perp/l])}\ \begin{array}{l}\mathrm{Nuova}(l,r,s)\\ l \in Loc_T\end{array} \qquad \frac{[\![e]\!](r,s) = (v,s')}{[\![t\ x = e]\!](r,s) = (r[l/x],s'[v/l])}\ \begin{array}{l}\mathrm{Nuova}(l,r,s')\\ l \in Loc_T\end{array}$$

$$\frac{[\![e]\!](r,s) = (v,s') \qquad [\![es]\!](r,s') = (lv,s'')}{[\![e\ \ es]\!](r,s) = (v\ lv,s'')} \qquad \frac{[\![e]\!](r,s) = (v,s')}{[\![x = e]\!](r,s) = (v,s'[v/r(x)])}$$

$$\frac{}{[\![x]\!](r,s) = (s(r(x)),s)}\ r(x) \in Loc \qquad \frac{[\![es]\!](r,s) = (lv\ ,s')}{[\![f(es)]\!](r,s) = r(f)(lv,s')}$$

$$\frac{[\![e]\!](r,s) = (v,s') \qquad [\![e']\!](r,s') = (v',s'')}{[\![e + e']\!](r,s) = (a,s'')}\ v + v' = a$$

etc...

Regole della Sem. Denot. per Stats e Prog

$$\frac{[\![st]\!](r,s) = s' \qquad [\![sts]\!](r,s') = s''}{[\![st\ \ sts]\!](r,s) = s''}$$

$$\frac{[\![e]\!](r,s) = (v\ ,s')}{[\![e;]\!](r,s) = s'}$$

$$\frac{[\![e]\!](r,s) = (tt\ ,s') \quad [\![sts]\!](r,s') = s''}{[\![\mathbf{if}\ (e)\ \{sts\}\ \mathbf{else}\ \{sts'\}]\!](r,s) = s''}$$

$$\frac{[\![e]\!](r,s) = (ff\ ,s') \quad [\![sts']\!](r,s') = s''}{[\![\mathbf{if}\ (e)\ \{sts\}\ \mathbf{else}\ \{sts'\}]\!](r,s) = s''}$$

$$\frac{[\![e]\!](r,s) = (ff\ ,s')}{[\![\mathbf{while}\ (e)\ \{sts\}]\!](r,s) = s'}$$

$$\frac{[\![e]\!](r,s) = (tt\ ,s') \qquad [\![sts]\!](r,s') = s'' \qquad [\![\mathbf{while}\ (e)\ \{sts\}]\!](r,s'') = s_0}{[\![\mathbf{while}\ (e)\ \{sts\}]\!](r,s) = s_0}$$

$$\frac{[\![ds]\!]([\,],(i,\lambda,[\,])) = (r\ ,s) \qquad [\![sts]\!](r,s) = (i',o,m)}{[\![ds\ \mathbf{main()}\ sts]\!](i) = o}$$