First link with MARIO: need for stability

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Workshop coordinates:

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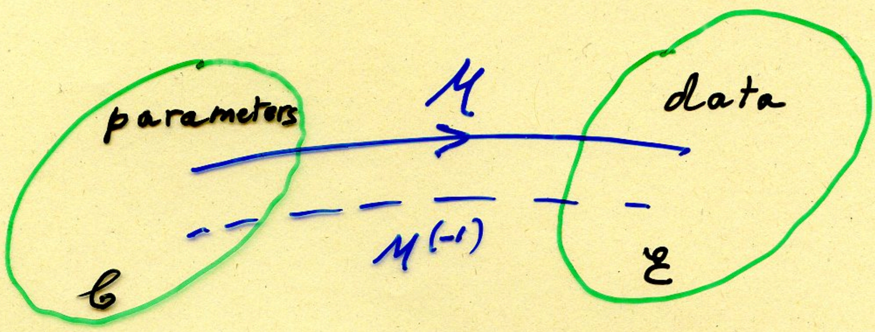
"nel mezzo del cammin di nostra vita"

En souvenir admiratif du premier travail présenté par Mario (en coll. avec Christine et Viano) à la rencontre Problèmes Inverses de 1977 à Montpellier.

(1)

Looks at STABILITY: POINTS ① ② ③ ④

1) **I.P.**



In I.P. stability is essentially continuity of the chosen $M^{(-1)}$; it depends on topologies in $\mathcal{P}$ and $\mathcal{E}$. They shoul be consistent with phyrics. I was more intereste by resolution.

2) **Mech.** $\underline{x} = \{x_i\}$ $i=1,\dots n$ = displacement of perturbed motion from the initial one

$$\frac{\partial x_i}{\partial t} = f_i(\underline{x}, t) \quad i = 1, \dots n \qquad \text{continuous in } \mathcal{D} \ni (\underline{0}) \text{ for } t \geq t_0$$

Stability: $(\forall \varepsilon)(\varepsilon > 0)(\exists \delta)(\delta > 0)(\operatorname{Sup} |x_i(t_0)| < \delta) \Rightarrow$

$\Rightarrow (\operatorname{Sup} |x_i| < \varepsilon)(t \geq t_0)$ (Liapunov)

3) **SIGNAL** S. is "stable" if structure kept in the propagation

4) **Modeling** Modeling a phenomenon which evolves with time is "stable" as long (and only as long as) all its parameters evolve in a continuous way.

NOTICE I.P. are tractable only in the model range

MODELING PROCESS

A0 General description and a priori knowledge on parameters

A1 Conservation laws. Boundary conditions

→ General Equations

B0 Working assumptions define ranges of reliability for parameters such that models can be constructed.

B1 Continuity assumptions in these ranges enable to make models "tractable" either by analytic developments or by numerical ones

⟹ and the ranges of stability for these models must be checked!

My own studies related to stability

In all my prof. life, I was attracted

- either by what I felt difficult to understand
- or by what was obviously misunderstood by dogmatic coll.

Point 1 I did not work on it, so beautifully settled by Mario et al., because I felt more attractive their points on degrees of freedom and resolution. Only one, I reacted to dogmatic statements of stupid mathematicians : In the I.S.P. for Schrödinger operator on $\mathbb{R}$

$$\left[-\frac{\partial^2}{\partial x^2} + V(x)\right] f(k,x) = k^2 f(k,x)$$

where data on the continuous spectrum is the reflection coefficient $R^+(k)$, no information on the discrete spectrum and the DOGMATIC STATEMENT claims for solution uniqueness if V has finite support.

- True, but I proved (Inv. Ill Posed Pbs 1996) that allowing the smallest perturbation on data and on the supposedly zero tail outside of the support generates STRONG Non Uniqueness and Instabilities !

Point 2

My current study (2011) is on the case of integrable nl p de and of l p de whose one or two coefficients are not constant but given solutions of an integrable nl p de. For instance, let V be a solution of KdV:

$$\frac{\partial V}{\partial t} + \frac{1}{4} V''' - \frac{3}{2} V V' = 0 \qquad \text{KdV}$$

the associated l p de can be (for instance)

$$\frac{\partial a}{\partial t} + \frac{1}{4} a''' - \frac{3}{4} (aV)' = 0 \qquad \text{LKdV}$$

It turns out that it can be fairly completely solved with help of the spectrum associated to KdV, ie the Schrödinger one! (GIST, introduced by me few years ago J. Nl math phys, 2005, Inv Prob. 2006)

⇒ We can study stability conditions for either motion (KdV or LKdV) as Liapounov did in classical mechanics (work in progress for multisolitons or linear "clones" of them)

Point 3 (signal stability)

It was the long time dream of Louis de Broglie to show "signal" gathering the properties of a wave and those of a particle.

[I hardly remember his remarks (I was a 20 year child, he was very elegant, refined french, high looks)]

Math. Physicists began to consider this idea when looking for 3d solitons. So I did when I studied in 1990 a new class of n.l. Schrödinger equations, 3d, homogeneous, time reversal invariant, norm cons, c. of mass separating :

$$i\frac{\partial \Psi}{\partial t} = (-\Delta + V)\Psi + s\Psi\,\Delta(\text{Log}\,|\Psi|) \qquad (1)$$

$(s \in \mathbb{R})$

$$i\frac{\partial \Psi}{\partial t} = (-\Delta + V)\Psi + s\Psi\,\Delta(|\Psi|)/|\Psi| \qquad (2)$$

$\Rightarrow$ The first one has exponentially confined solutions if $s > 1$ **but** it cannot be derived from a local lagrangian density!

The second one does not propagate solitary wave **but** can be derived from local lagrangian.

In 1994, G. Auberson and me proved that the confined solutions of (1) are **unstable**

Point 4: Model Instabilities

I studied them only long time ago, in the course of engineering works:

A)Slide tsunamis

I missed time to show clearly the corresponding transparents during the workshop and I will not do it here. To be short, I have used the stability of the complicated nonlinear direct problem of wave generation to get simple approximate results, and the dependence on main parameters, if the slope is not too large. Strong instabilities, and breaks, making necessary other models, appear if the slope is out of this range, or after a long propagation. For details, see P.C. Sabatier Journal of Fluid Mechanics, Vol. **126**, 27-58 (1983) and ch. XVII, pp.723-759 in Vol. VI of Encyclopedia of Fluid Mechanics. N. Cheremesinoff Ed., Gulf Publ. Houston, London, Paris, Tokyo (1986).

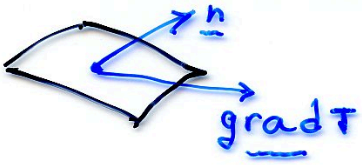
B)Design of a passive solar house, "zero energy" for week-ends.

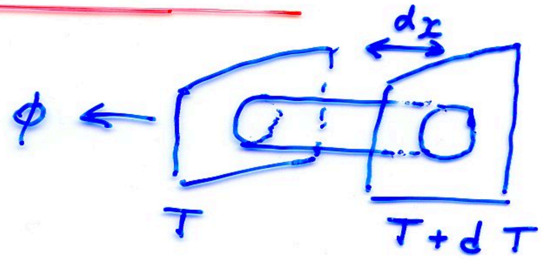
It was the 3rd one built in France (1977), the only one on seaside (it can hardly be more close). The goals were:
During winter, find always more than 14 degrees in all rooms when you go into the house, so that, if necessary, a fire of (floated) woods in an open chimney can get rapidly to 20 degrees.
During summer, the house should be pleasant, never too warm.
Do not use solar cells or electrical air-conditionning.
These goals were reached, thanks to a design shown in tranparents, which uses partiallly the solar-chimney wall invented by F.Trombe, but also several original ideas for the external insolation made of foamglass, for using the sea water (30 cm below the basement) in order to cool, and with a good study of heat and humidity transfers. In 34 years, only two instabilities:

Two weeks below 14 degrees in 1981, minimum eight, due to the conjonction of three unlikely events; two weeks without sun, sea exceptionnaly cold, a very stormy depression and no defence against it inside the chimney (we put one later). Only one weather-connected instablity in more than 30 years is not too much!
In may 2008, the well-known hooligans brain instability, and the fact that the city mayor lives in a neighbouring house, led them to break the solar glasses, as it is seen in transparents. Of course, it is now repaired.

Some reminders

CONDUCTION





Fourier Law $\phi = -\lambda\, \underline{n} \cdot \underline{grad}\, T$

flux — thermal conductivity — temperature gradient

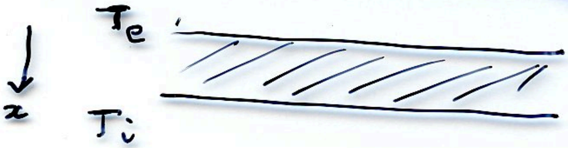
⇒ (1) Permanent state and (almost) invariant λ

$$\lambda \Delta T + W = 0$$

W = power

In particular, if w = 0, "architect rules" $dT/dx = 0$ (walls)

(2) Periodic attack of a wall ("cellar rules")



$$T_e = T_0 + T_1 \exp[i\omega t]$$

$$\Rightarrow T(t,x) = T_0 + T_1 \exp[-x/\ell]\, \exp[i(\omega t - x/\ell)]$$

$$\ell = \sqrt{2\lambda / \rho C \omega}$$

⇒ attenuation and phase-shift

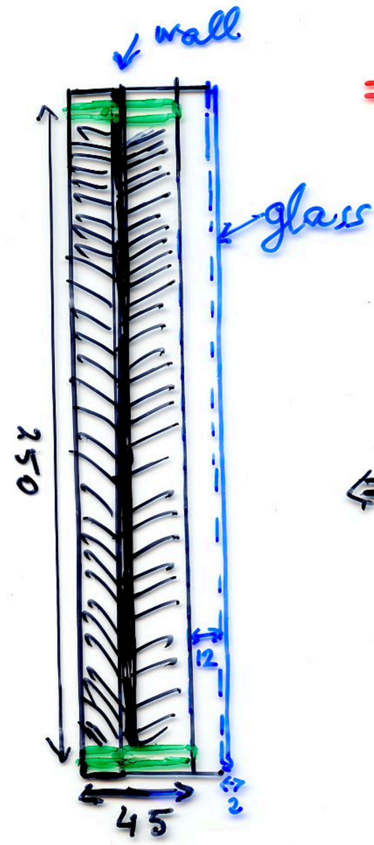
CONVECTION and water vapor transfers

fluid motions . $\dfrac{\text{vapor}}{\text{rate of air renewal}} \sim 2 \to 8\ g/m^3$

Condensation – { on walls / inside isolant

Examples of ph. values

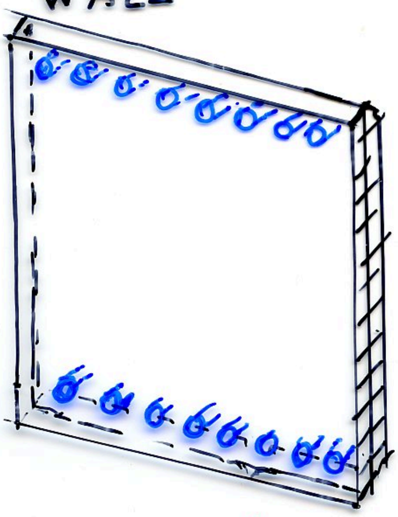
	λ (w m °C)	Permeability (g.m. h mm Hg)
polystyrene	<~ 0,04	~ 10^{-3}
foamglass	≈ 0,045	~ 10^{-7}

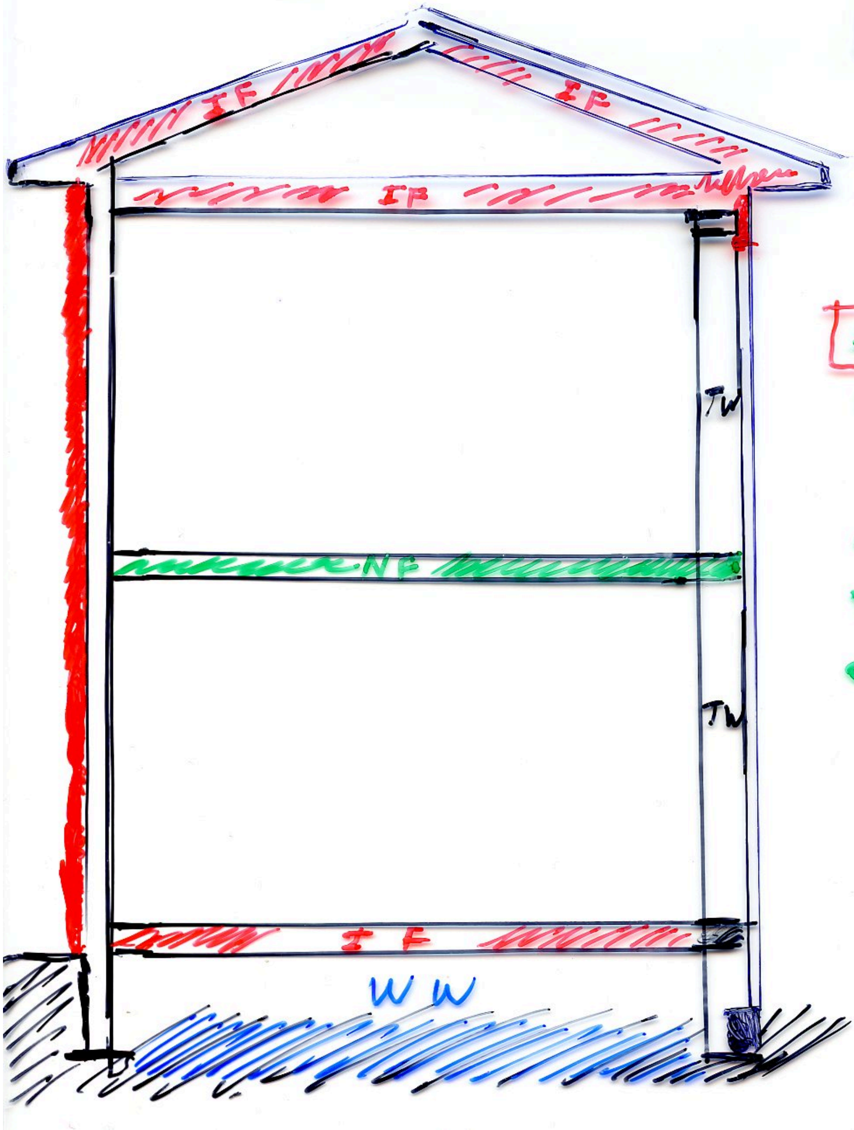


⇒ S/SE

WINTER: During day, solar chimney behind glasses heats the room by convection through the holes.
During night, heating thanks to the 12 hours phase-shift, (and 2x6 h. phase shift on all other walls)

⇐ TROMBE WALL





SUMMER Cooling thanks to the humid basement (sea water level is only 30 cm below WW, and good transfer of humidity.