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Stability Problems in Inverse Diffraction

MARIO BERTERO AND CHRISTINE DE MOL

Abstract—**Inverse diffraction consists in determining the field distribution on a boundary surface from the knowledge of the distribution on a surface situated within the domain where the wave propagates. This problem is a good example for illustrating the use of least-squares methods (also called regularization methods) for solving** *linear ill-posed inverse problems.* **We focus on obtaining error bounds for regularized solutions and show that the stability of the restored field far from the boundary surface is quite satisfactory: the error is proportional to $\epsilon^{\alpha}(\alpha \simeq 1)$, ϵ being the error in the data (*Hölder continuity*). However, the error in the restored field on the boundary surface is only proportional to an inverse power of $|\ln \epsilon|$ (*logarithmic continuity*). Such a poor continuity implies some limitations on the resolution which is achievable in practice. In this case, the resolution limit is seen to be about half of the wavelength.**

I. INTRODUCTION

ILL-POSEDNESS is a typical feature of many linear inverse problems: an arbitrarily small perturbation of the data can produce an arbitrarily large variation of the solution. Since experimental data are always affected by noise, we need methods for restoring numerical stability. This can be achieved by means of supplementary constraints expressing some prior knowledge about the solution. In the case of quadratic constraints, an estimate of the solution is provided by least-squares methods (also called *regularization methods*) [1]-[3]. One says that stability has been restored if the error in the least-squares solution tends to zero when the error in the data tends to zero.

As a good example for discussing regularization methods, we will consider the inverse diffraction problem formulated in a very simple geometry, i.e., plane surfaces [4], [5]. Then the *direct problem* is the following: find a function $u(x, y, z)$ satisfying the reduced wave equation in the half-space $z \geqslant 0$ and, taking given values on the boundary plane (sometimes called the source plane) $z = 0$,

$$\nabla^2 u + k^2 u = 0 \qquad u(x, y, 0) = f(x, y). \tag{1}$$

In addition, it is required that u satisfies the Sommerfeld radiation condition at infinity [6], which ensures that u represents at infinity an outgoing spherical wave and is essential for the uniqueness of the solution of problem (1). The existence of the solution of the direct problem can also be proved under rather mild conditions on f [7]. The solution u is then given by

$$\begin{aligned} u(x, y, z) &= (K_z * f)(x, y) \\ &= \iint_{-\infty}^{+\infty} K_z(x - x', y - y')f(x', y')\,dx'\,dy' \end{aligned} \tag{2}$$

$$K_z(x, y) = -\frac{1}{2\pi}\frac{\partial}{\partial z}\frac{e^{ikr}}{r}, \qquad r = (x^2 + y^2 + z^2)^{1/2}$$

and it can also be represented as an angular spectrum of plane waves:

$$\begin{aligned} u(x, y, z) &= \iint_{-\infty}^{+\infty} F(p, q)e^{ik(px+qy+mz)}\,dp\,dq, \quad z > 0 \\ F(p, q) &= \left(\frac{k}{2\pi}\right)^2 \iint_{-\infty}^{+\infty} f(x, y)e^{-ik(px+qy)}\,dx\,dy \\ m &= (1 - p^2 - q^2)^{1/2}, \quad M = M(p, q) \\ &= \operatorname{Im}(1 - p^2 - q^2)^{1/2} \geqslant 0. \end{aligned} \tag{3}$$

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It clearly results from (3) that a small perturbation of f produces a small perturbation of the solution u. Hence the direct problem is well-posed: there exists a unique solution which depends continuously on the data.

More difficult is the *inverse diffraction problem*: determine the field u in the region $0 \leqslant z < a$ from the knowledge of the field in the plane $z = a$. Clearly, it is sufficient to determine $f(x, y)$, i.e., the field in the boundary plane $z = 0$, or else the spectral amplitude $F(p, q)$. Now, the inverse diffraction problem is ill-posed. This can easily be seen by considering, for instance, the following field distribution in the half-space $z > 0$:

$$u(x, y, z) = \exp\left[-\sqrt{p} + ik(px + y) + kp(a - z)\right]. \tag{4}$$

By taking p sufficiently large, we get a field which is arbitrarily small for $z \geqslant a$, and arbitrarily large in the region $0 \leqslant z < a$. Hence small errors in the data function can be considerably amplified in the region $0 \leqslant z < a$. This example also indicates that ill-posedness is due to the effect of the so-called inhomogeneous (evanescent) waves ($p^2 + q^2 > 1$).

In Section II we consider least-squares (regularized) solutions of the inverse diffraction problem, we show the relationship between regularization methods and numerical filtering [8], and we discuss the stability of the solutions. In particular, when imposing a bound on the field in the plane $z = 0$, the error in the restored field in the plane $z = b < a$ is proportional to ϵ^α, $\alpha = b/a > 0$, where ϵ is the error in the data (Hölder continuity). In order to get stability up to the boundary plane $z = 0$, we need bounds also on the derivates of the field, and in this case, the error in the solution is roughly proportional to an inverse power of $|\ln \epsilon|$ (*logarithmic continuity*). Such stability is not necessarily enough in practice; a very small error in the data can still induce a rather large error in the solution and prevent accurate numerical computations. Even a lowering of the noise by many orders of magnitude will not improve significantly the solution. We will see that this fact is related to the existence of a certain resolution limit, measuring the size of the finest details which can be restored with an acceptable accuracy. Such a feature would be intrinsic to each ill-posed problem which is solved by least-squares methods and affected by logarithmic continuity [9]-[11]. In Section III we give a way for estimating the resolution limit, showing that for our diffraction problem, it is approximately $\lambda/2$, where $\lambda = 2\pi/k$ is the wavelength of the radiation field.

II. STABILITY OF LEAST-SQUARES SOLUTIONS

Assuming that the noise can be described by an additive term $e(x, y)$, let us write the data, i.e., the noisy field distribution in the plane $z = a > 0$, as follows:

$$g(x, y) = (K_a * f)(x, y) + e(x, y) \tag{5}$$

where $f(x, y)$ is the unknown field in the plane $z = 0$. Now, there does not exist in general any solution of the reduced wave equation in the region $0 \leqslant z < a$, which represents radiation traveling in the positive z direction and taking the given values $g(x, y)$ in the plane $z = a$. Indeed, because of the smoothing effect due to evanescent waves, such a solution exists only if g has some peculiar analyticity properties, which are in general not satisfied by the noise function $e(x, y)$. Therefore, the best we can do is to look for a field in the region $0 \leqslant z <$ a which approximates the data function $g(x, y)$ within the experimental errors. If we assume that $|e(x, y)|^2$ is integrable and that ϵ^2 is an upper bound on its integral, then using the field representation (3) and the Parseval equality for the Fourier transform, we get the following constraint on the spectral amplitude $F(p, q)$:

$$\left(\frac{2\pi}{k}\right)^2 \iint_{-\infty}^{+\infty} |F(p, q)e^{ikam} - G(p, q)|^2 \, dp\, dq \leqslant \epsilon^2, \tag{6}$$

$G(p, q)$ being the Fourier transform of $g(x, y)$. Since the factor $\exp(ikam)$ is exponentially decreasing when $p^2 + q^2 > 1$, condition (6) does not exclude solutions which contain in a significant way strongly oscillating terms like (4). In order to control these terms and restore the stability, we have to assume some constraint on the solution, for instance, the following bound on $F(p, q)$:

$$\left(\frac{2\pi}{k}\right)^2 \iint_{-\infty}^{+\infty} |F(p, q)|^2 \, dp\, dq \leqslant E^2 \tag{7}$$

where E is a prescribed constant.

At this point, any spectral amplitude $F(p, q)$ satisfying both constraints (6) and (7) can be taken as a solution of the inverse diffraction problem. Then it is more convenient to combine these two constraints into the single one

$$\left(\frac{2\pi}{k}\right)^2 \iint_{-\infty}^{+\infty} \left\{ |F(p, q)e^{ikam} - G(p, q)|^2 + \left(\frac{\epsilon}{E}\right)^2 |F(p, q)|^2 \right\} dp\, dq \leqslant \epsilon^2. \tag{8}$$

According to regularization theory [1], [2] the spectral amplitude $\tilde{F}(p, q)$ which minimizes the functional at the left side of (8) is taken as an estimate of the solution of the inverse problem.

It is not difficult to show that $\tilde{F}(p, q)$ can be obtained by means of a filtering of the data:

$$\tilde{F}(p, q) = e^{-ikam} H(p, q) G(p, q) \tag{9}$$

where the filter function given by $H(p, q) = [1 + (\epsilon/E)^2 \exp(2kaM)]^{-1}$ ($M = M(p, q)$ is defined in (3)). Clearly, the effect of the Fourier components of g, with $\epsilon^2 \exp(2kaM) \gg 1$, is now suppressed. In this case, regularization is equivalent to numerical filtering as discussed by Twomey [8]. Because of the filtering, the estimated field distribution in the plane $z = 0$, i.e., $\tilde{f}$, is very smooth (analytic).

The restored field $\tilde{u}(x, y, z)$ in the region $0 \leqslant z < a$ is given by (3), with $F(p, q)$ replaced by $\tilde{F}(p, q)$. Using the techniques described in [11], it can be proved that

$$\rho(\epsilon, E; b) = \max \left(\iint_{-\infty}^{+\infty} |\tilde{u}(x, y, b) - u(x, y, b)|^2 \, dx\, dy \right)^{1/2} \leqslant E \left(\frac{\epsilon}{E}\right)^\alpha \tag{10}$$

where $\alpha = b/a < 1$; the maximum is taken over the class of field distributions associated to the function $F(p, q)$ and $G(p, q)$ satisfying the constraint (8). The quantity $\rho(\epsilon, E; b)$, which is in fact a modulus of continuity, gives a measure of the error in the solution for a given error level in the data. The upper bound (10) tends to zero, when $\epsilon \to 0$, as a power of ϵ (Hölder continuity). Hence the stability of the solution is quite good when the ratio b/a is not small, but becomes rather bad when $b \to 0$. In the boundary plane $z = 0$ itself, the constraint (7) does not restore the stability. A similar feature arises for the problem of analytic continuation of a function from a smaller concentric circle into the unit disk [12]. In order to get stability up to the boundary (here the plane $z = 0$), we have to prescribe a bound also on the derivatives of the solution. More precisely, if we require that

$$\left(\frac{2\pi}{k}\right)^2 \iint_{-\infty}^{+\infty} k^2(p^2 + q^2)|F(p, q)|^2 \, dp \, dq \leq E^2 \tag{11}$$

(i.e., a bound on the integral of $|\nabla f(x, y)|^2$), and if we combine again the constraints (6) and (11) into a single one like (8), we get an estimate of the angular spectrum like (9), the filter function being now $H(p, q) = [1 + (k\epsilon/E)^2(p^2 + q^2) \exp(2kaM)]^{-1}$. Besides, it is not difficult to prove, by means of standard techniques [11], that in this case

$$\rho(\epsilon, E; b = 0) \cong aE \left| \ln\left(\frac{k\epsilon}{E}\right) \right|^{-1}, \qquad \epsilon \to 0. \tag{12}$$

Hence we have restored the stability also in the boundary plane, but it is rather poor (logarithmic continuity).

In the previous considerations, we have always assumed the unknown spectral amplitude $F(p, q)$ to be square integrable. In many relevant cases, however, it is important to consider spectral amplitudes of the form [13]: $F(p, q) = V(p, q)/m$. If $V(p, q)$ is not zero for $p^2 + q^2 = 1$, then $F(p, q)$ is not square integrable in a neighborhood of the unit circle, but then we can require, for instance, that $V(p, q)$ be square integrable, and we can prescribe the following constraint:

$$\left(\frac{2\pi}{k}\right)^2 \iint_{-\infty}^{+\infty} |ikmF(p, q)|^2 \, dp \, dq \leq E^2. \tag{13}$$

Note that $ikmF(p, q)$ is just the Fourier transform of the normal derivative of the field distribution in the plane $z = 0$. In this case too, only logarithmic continuity is obtained for the least squares solutions.

III. RESOLUTION LIMITS

When logarithmic continuity arises, the accuracy of the solution cannot be significantly improved by a lowering of the noise by many orders of magnitude. Therefore, one expects the existence of some resolution limit, giving the size of the smallest details which can be restored. This resolution limit should be quasi-insensitive to the actual noise level in the data, and so, intrinsic to the problem (at least in the frame of least-squares methods). The following procedure allows us to check this feature and to estimate, for a given problem affected by logarithmic continuity, the corresponding characteristic resolution limit [10], [11].

Let us consider again the constraint (8) and put for simplicity $E = 1$. For any field distribution $f(x, y)$ in the plane $z = 0$, satisfying (8), we define $f_\sigma(x, y)$ by

$$f_\sigma = w_\sigma * f, \; w_\sigma(x, y) = (\sqrt{2\pi}\sigma)^{-2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right). \tag{14}$$

The function $f_\sigma(x, y)$ is obtained by blurring the actual solution $f(x, y)$, according to the weight function w_σ. In the same way, we define $\tilde{f}_\sigma = w_\sigma * \tilde{f}$. Then for any f satisfying (8), one can prove that

$$|\tilde{f}_\sigma(x, y) - f_\sigma(x, t)| \leq C(\sigma)\, \epsilon \tag{15}$$

where $C(\sigma)$ is independent of the variables x, y. In other words, when the noise vanishes, the blurred least-squares estimate $\tilde{f}_\sigma$ converges uniformly to the blurred solution f_σ. At first glance, this convergence seems to be excellent (Hölder continuity), but it must be pointed out that $C(\sigma) \to +\infty$ when $\sigma \to 0$: there is a degradation of the stability when the weighting function $w_\sigma(x, y)$ becomes very narrow.

This fact suggests a way for analyzing resolution limits. Indeed, the error in the restoration of $f_\sigma(x, y)$ can be interpreted as the error in the restoration of details whose size is of the order of σ, in a neighborhood of the point $P = \{x, y\}$. Now, since the absolute error (15) tends to infinity when $\sigma \to 0$, it is better to look for a suitable definition of relative errors.

Let us define the following modulus of continuity:

$$\rho(\epsilon, a; \sigma) = \max |\tilde{f}_\sigma(x, y) - f_\sigma(x, y)|. \tag{16}$$

The maximum is taken not only over all field distributions f compatible with the constraint (8) for a given data g, but also over all possible data g (i.e., all $\tilde{f}$). Since the problem is invariant for translations, the maximum does not depend on the variables x, y. The quantity (16) represents the largest error which can be committed, and this might be too pessimistic in some practical cases. Error estimates corresponding to a particular data g can also be obtained [10]. $\rho(\epsilon, a; \sigma)$ can be computed by means of general results about blurred solutions (see [2, lemma 5] and [10]). On the other hand, let us define the quantity

$$\rho(\sigma) = \max \; |f_\sigma(x, y)| \tag{17}$$

where f varies in the class defined by the constraint (7) (with $E = 1$). $\rho(\sigma)$ is the maximum variation of $|f_\sigma(x, y)|$ allowed by the constraint (7). Then a reasonable definition of relative errors is the following [10], [11]:

$$\delta(\epsilon, a; \; \sigma) = \frac{\rho(\epsilon, a; \sigma)}{\rho(\sigma)}. \tag{18}$$

If we introduce the parameters $\alpha = a/\lambda$ and $\eta = \sigma/\lambda$, where $\lambda = 2\pi/k$ is the wavelength of the radiation field, then by computing $\rho(\epsilon, a; \sigma)$ and $\rho(\sigma)$, we get the following result:

$$\delta(\epsilon, \alpha; \eta) = \sqrt{2}\epsilon(2\pi\eta)\left(\int_0^{+\infty} \frac{e^{-4\pi^2\eta^2\rho^2}}{e^{-4\pi\alpha M(\rho)} + \epsilon^2}\, \rho\, d\rho\right)^{1/2} \tag{19}$$

where $M(\rho) = 0$ if $\rho \leqslant 1$ and $M(\rho) = (\rho^2 - 1)^{1/2}$ if $\rho > 1$. Equation (19) can also be written as follows:

$$\delta(\epsilon, \alpha; \eta) = \left[\frac{e^{-4\pi^2\eta^2} + \epsilon^2}{1 + \epsilon^2} - e^{-4\pi^2\eta^2} I\left(\epsilon, \frac{\alpha}{\eta}\right)\right]^{1/2} \tag{20}$$

where

$$I(\epsilon, \xi) = 2\int_0^{+\infty} \frac{e^{-(2\xi t + t^2)}}{e^{-2\xi t} + \epsilon^2}\, t\, dt. \tag{21}$$

Since $I(0, \xi) = 1$ and $I(\epsilon, +\infty) = 0$, it follows that

1) $\delta(0, \alpha; \eta) = 0$ for $\alpha < +\infty$, $\eta > 0$ (stability for blurred restoration);
2) $\delta(\epsilon, \alpha; 0) = 1$ (one hundred percent error for pointwise restoration);
3) $\delta(\epsilon, +\infty; \eta) = [\exp(-4\pi^2\eta^2) + \epsilon^2]^{1/2}/(1 + \epsilon^2)^{1/2}$ (relative error in the case of far-field data, i.e., no knowledge about evanescent waves).

As a consequence of 3), we can say that the second term at the right-hand side of (20) represents the contribution of evanescent waves. This contribution is essential for stability (see property 2); however, for reasonable values of ϵ (since $E = 1$, ϵ^{-1} is a sort of signal-to-noise ratio), i.e., $\epsilon = 10^{-2} \div 10^{-4}$, and for a distance of a few wavelengths between the data plane and the boundary plane, i.e., $\alpha \cong 10$, this contribution is practically negligible. In Fig. 1 we give the results of a numerical computation of $\delta(\epsilon, \alpha; \eta)$, using (20), (21). The undotted line represents the asymptotic value $\delta(\epsilon, +\infty; \eta)$ for $\epsilon = 10^{-2}$; as we see, $\delta(\epsilon, +\infty; \eta)$ reaches the value 10^{-2} for $\eta \cong 0.5$, i.e., $\sigma \cong \lambda/2$. The curve for $\delta(\epsilon, +\infty; \eta)$ with $\epsilon = 10^{-4}$ is not drawn since it does not practically depart from the curve with $\epsilon = 10^{-2}$ up to $\eta \cong 0.4$; the value 10^{-4} is attained for $\eta \cong 0.6$. The dotted curves correspond to the case where the data are given on a plane at a distance of one wavelength from the boundary plane. It is seen that there is no significant improvement with respect to the asymptotic case; moreover, the relative error is not very sensitive to the actual value of ϵ.

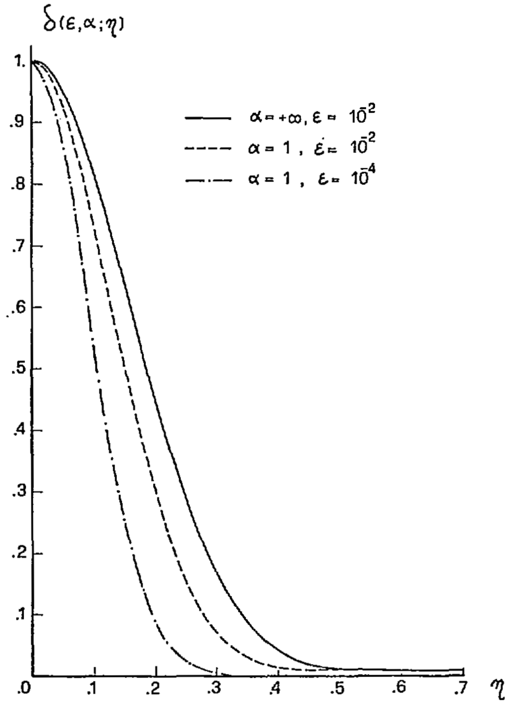

Fig. 1. Relative error in restoration of blurred solutions versus resolution parameter, for various values of noise level ϵ and of distance α from boundary plane.

IV. CONCLUSION

In the previous sections, we have considered the case of a scalar field; the generalization to electromagnetic vector fields is straightforward. Indeed, as is well-known, the electromagnetic field in the half-space $z > 0$ is determined by the tangential components of the electric field in the boundary plane $z = 0$. Other geometries, like cylindrical and spherical surfaces, can be treated in a similar way, using the appropriate expansions.

The property of logarithmic continuity arises for many linear ill-posed inverse problems when using, for solving them, the basic available tool, i.e., least-squares methods. In fact, this pathology is closely related to the smoothing effect which appears in the corresponding direct problem. For instance, in the problem discussed in this paper, because of evanescent waves, the field becomes smoother and smoother when propagating further and further from the source plane $z = 0$. One expects that the stronger the smoothing effect will be in the direct problem, the more difficult the inverse problem becomes and the worse the restored continuity. Indeed too much information about the solution is then completely lost in the data. Many examples of inverse problems affected by logarithmic continuity are discussed in [11]; let us just state here the solution of first-kind Fredholm integral equations with analytic kernels (and in particular, deconvolution problems), the problem of bandwidth extrapolation, the continuation of an analytic function up to the boundary of the analyticity domain, the Cauchy problem for the reduced wave equation. In all these cases, the method sketched in Section III can be used more generally for estimating the resolution limit intrinsic to the least-squares solutions of a given problem. Resolution curves similar to those of the above figure have been obtained for a few other problems [10], [11]; because of logarithmic continuity, these curves are not very sensitive to the actual value of the signal-to-noise ratio.

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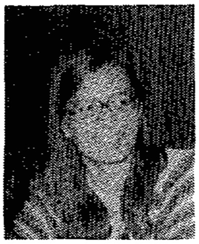

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