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Corrigendum: Efficient gradient projection methods for edge-preserving removal of Poisson noise

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Abstract

This file contains a complete proof of lemma 1 of the paper.

(Some figures may appear in colour only in the online journal)

The proof of lemma 1, given in appendix A of the paper [1] is incomplete because the upper bound (A.10) for $e_4(\xi)$ is correct only for $\xi \geq 0$ and therefore it provides an upper bound on the positive part of the expected value of the residual. No estimate of the negative part is given. Moreover, an attempt of completing the proof has shown that the approach proposed in [1] can be simplified. Since the lemma is the basis of a discrepancy principle for the reconstruction of Poisson data, introduced by the authors in that and subsequent papers [2, 3], we believe that it may be useful to provide a complete proof.

Lemma 1. Let Y_λ be a Poisson r.v. with expected value λ and consider the function of Y_λ defined by

$$F_\lambda(Y_\lambda) = 2 \left\{ Y_\lambda \ln \left(\frac{Y_\lambda}{\lambda} \right) + \lambda - Y_\lambda \right\}. \quad (1)$$

Then the following asymptotic estimate of the expected value of $F_\lambda(Y_\lambda)$ holds true

$$E\{F_\lambda(Y_\lambda)\} = 1 + O\left(\frac{1}{\lambda}\right), \quad \lambda \rightarrow +\infty. \quad (2)$$

Proof. Let us introduce a function $\phi(\xi)$ related to the values $F_\lambda(n)$, $n = 0, 1, \dots$ of the random variable $F_\lambda(Y_\lambda)$ as follows

$$F_\lambda(n) = 2 \left\{ n \ln \left(\frac{n}{\lambda} \right) + \lambda - n \right\} \doteq 2\lambda\phi(\xi) \quad (3)$$

where

$$\phi(\xi) = (1 + \xi) \ln(1 + \xi) - \xi, \quad \xi = \frac{n - \lambda}{\lambda} \in [-1, +\infty),$$

and $z \ln(z) = 0$ if $z = 0$. Then, from third order Taylor formula for $\ln(1 + \xi)$ we obtain

$$\phi(\xi) = \frac{1}{2}\xi^2 - \frac{1}{6}\xi^3 + \frac{1}{3}\xi^4 + (1 + \xi)e_3(\xi), \quad e_3(\xi) = \int_0^\xi \frac{(t - \xi)^3}{(1 + t)^4} dt \quad (4)$$

the residual $e_3(\xi)$ being a negative-valued function with a logarithmic singularity at $\xi = -1$. If we insert this expression in (3), we have

$$F_\lambda(n) = \frac{(n - \lambda)^2}{\lambda} - \frac{1}{3} \frac{(n - \lambda)^3}{\lambda^2} + \frac{2}{3} \frac{(n - \lambda)^4}{\lambda^3} + 2n e_3\left(\frac{n - \lambda}{\lambda}\right). \quad (5)$$

By considering the central moments of the Poisson random variable Y_λ , $\mu_k = E\{(Y_\lambda - \lambda)^k\}$, $k = 1, 2, \dots$, and taking into account that $\mu_2 = \mu_3 = \lambda$, $\mu_4 = 3\lambda^2 + \lambda$, we obtain

$$E\{F_\lambda(Y_\lambda)\} = \sum_{n=0}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} F_\lambda(n) = 1 + \frac{5}{3\lambda} + \frac{2}{3\lambda^2} + 2\mathcal{E}(\lambda), \quad (6)$$

where

$$\mathcal{E}(\lambda) \doteq E\left\{Y_\lambda e_3\left(\frac{Y_\lambda - \lambda}{\lambda}\right)\right\} = \sum_{n=1}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} n e_3\left(\frac{n - \lambda}{\lambda}\right). \quad (7)$$

The proof of the lemma is based on a suitable estimate of $\mathcal{E}(\lambda)$ when $\lambda \rightarrow +\infty$. $\square$

To this purpose we split the series into two parts: a partial sum with n ranging from 1 to $\lfloor \lambda/2 \rfloor$ (corresponding to $\xi \in (-1, -1/2]$) and a series with n ranging from $\lfloor \lambda/2 \rfloor + 1$ to $+\infty$ (corresponding to $\xi \in (-1/2, +\infty)$). As concerns the partial sum, we first observe that, since $0 \leq t - \xi < t + 1$,

$$(1 + \xi)|e_3(\xi)| \leq (1 + \xi) \int_\xi^0 \frac{dt}{1 + t} = -(1 + \xi) \ln(1 + \xi) \leq \frac{1}{e}; \quad (8)$$

next, we bound the sum by exploiting this inequality and, by remarking that $\lambda^m/m! < \lambda^n/n!$ when $m < n \leq \lfloor \frac{\lambda}{2} \rfloor$, we further bound the sum by the last term multiplied by the number of terms

$$\left| \sum_{n=1}^{\lfloor \frac{\lambda}{2} \rfloor} \frac{e^{-\lambda} \lambda^n}{n!} n e_3\left(\frac{n - \lambda}{\lambda}\right) \right| \leq \frac{\lambda}{e} e^{-\lambda} \left\lfloor \frac{\lambda}{2} \right\rfloor \frac{\lambda^{\lfloor \frac{\lambda}{2} \rfloor}}{\lfloor \frac{\lambda}{2} \rfloor!}. \quad (9)$$

If we use the Stirling lower bound of the factorial

$$n! \geq \sqrt{2\pi} n^{n+\frac{1}{2}} e^{-n}, \quad (10)$$

we have the following upper bound

$$\left| \sum_{n=1}^{\lfloor \frac{\lambda}{2} \rfloor} \frac{e^{-\lambda} \lambda^n}{n!} n e_3\left(\frac{n - \lambda}{\lambda}\right) \right| \leq \frac{\lambda}{e\sqrt{2\pi}} e^{-\lambda + \lfloor \frac{\lambda}{2} \rfloor} \left\lfloor \frac{\lambda}{2} \right\rfloor^{\frac{1}{2}} \left(\frac{\lambda}{\lfloor \frac{\lambda}{2} \rfloor} \right)^{\lfloor \frac{\lambda}{2} \rfloor}. \quad (11)$$

Finally, if we remark that $\lfloor \lambda/2 \rfloor = O(\lambda/2)$, we obtain

$$\left| \sum_{n=1}^{\lfloor \frac{\lambda}{2} \rfloor} \frac{e^{-\lambda} \lambda^n}{n!} n e_3\left(\frac{n - \lambda}{\lambda}\right) \right| = O\left(\lambda^{3/2} \left(\frac{e}{2}\right)^{-\lambda/2}\right), \quad (12)$$

and therefore the partial sum tends to zero exponentially fast.

As concerns the series with $n \geq \lfloor \lambda/2 \rfloor + 1$, observing that in the expression of $e_3(\xi)$ we have $1 + t > 1/2$, by considering separately the cases $\xi < 0$ and $\xi > 0$ we find that in both cases we have

$$|e_3(\xi)| \leq 4\xi^4 \quad (13)$$

Figure 1. Plot of the function $E\{F_\lambda(Y_\lambda)\}$, defined in equation (6), as a function of λ . The asymptotic limit is approximately reached around $\lambda = 20$.

which implies

$$\begin{aligned}
 \left| \sum_{n=\lfloor \frac{\lambda}{2} \rfloor + 1}^{+\infty} \frac{e^{-\lambda} \lambda^n}{n!} n e_3\left(\frac{n-\lambda}{\lambda}\right) \right| &\leq 4 \sum_{n=0}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} n \left(\frac{n-\lambda}{\lambda}\right)^4 \\
 &= 4 \sum_{n=0}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} \left[\frac{(n-\lambda)^5}{\lambda^4} + \frac{(n-\lambda)^4}{\lambda^3} \right] \\
 &= 4 \left(\frac{\mu_4}{\lambda^3} + \frac{\mu_5}{\lambda^4} \right) = O\left(\frac{1}{\lambda}\right), \quad (14)
 \end{aligned}$$

since $\mu_5 = 10\lambda^2 + \lambda$. Thus, taking into account the equation (6), the proof of the lemma is completed.

Remark. In figure 1 we show the plot of the function $E\{F_\lambda(Y_\lambda)\}$ defined in equation (6). The plot shows that the function grows from zero up to a maximum value of approximately 1.1604 corresponding to $\lambda = 1.338$ and subsequently tends rapidly to 1, according to the result of the lemma.

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References

- [1] Zanella R, Boccacci P, Zanni L and Bertero M 2009 Efficient gradient projection methods for edge-preserving removal of Poisson noise *Inverse Problems* **25** 045010
- [2] Bertero M, Boccacci P, Talenti G, Zanella R and Zanni L 2010 A discrepancy principle for Poisson data *Inverse Problems* **26** 105004
- [3] Staglianó A, Boccacci P and Bertero M 2011 Analysis of an approximate model for Poisson data reconstruction and a related discrepancy principle *Inverse Problems* **27** 125003