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F Benvenuto[1], A La Camera[2], C Theys[3], A Ferrari[3], H Lantéri[3] and M Bertero[2]

[1] DIMA, Università di Genova, Via Dodecaneso 35, I 16146 Genova, Italy
[2] DISI, Università di Genova, Via Dodecaneso 35, I 16146 Genova, Italy
[3] Laboratoire Hippolyte Fizeau, Université de Nice Sophia Antipolis, CNRS UMR6525, 06108 Nice Cedex 2, France

E-mail: bertero@disi.unige.it

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The main result of section 2, namely the existence of maximum likelihood (ML) solutions of the problem of image reconstruction with data corrupted by Poisson and Gaussian noise, is correct but its derivation is partially erroneous and incomplete.

The first point is that lemma 1 cannot be used for proving the concavity of the likelihood function of the Poisson–Gaussian problem because the likelihood of the Poisson problem is not concave. Hence, the Poisson–Gaussian likelihood is also not. However, it is true that its negative logarithm is convex. For the convenience of the readers, we here rewrite this function:

$$J(x) = -\sum_{i\in S} \ln\left\{\sum_{n=0}^{+\infty} \frac{\mathrm{e}^{-(Hx+b)_i}(Hx+b)_i^n}{n!}\,\mathrm{e}^{-\frac{1}{2\sigma^2}(n-y_i)^2}\right\}, \qquad x \geqslant 0, \tag{1}$$

which is given in equation (4) of the paper.

The second proof of convexity, given in the paper, is correct but a proof of coercivity, which is required for proving the existence of minimizers of this function hence the existence of ML solutions, is not given, even if it is elementary. Here, we complete this point. In conclusion, the following result replaces propositions 1–3 of the paper.

Proposition 1. *The function $J(x)$ is nonnegative, convex and coercive, and strictly convex iff the equation $Hx = 0$ has the unique solution $x = 0$.*

Proof. Convexity and strict convexity follow from lemma 2 and the expression of the Hessian given in equation (12). As concerns the other properties, first remark that, for each $i \in S$, the corresponding series in the expression of $J(x)$ is uniformly convergent for $x \geqslant 0$ (it is bounded by a convergent series with constant terms) and takes a value in (0,1). Hence, non-negativity of $J(x)$ follows. Next, as shown, e.g., in [1] (see remark 1 of the paper), $||Hx||_2$ is coercive, as a consequence of the properties of the imaging matrix H given in equation (1), and therefore, for $x \to +\infty$, there exists at least one $i \in S$, such that $(Hx+b)_i \to +\infty$. Each term of the

corresponding series tends to zero and therefore, thanks to uniform convergence, the series also tends to zero, implying that $J(x)$ tends to infinity and therefore is coercive. □

We remark that a sufficient condition for uniqueness is the injectivity of the imaging matrix H, as in the case of Gaussian noise, while in the case of Poisson noise, strict positivity of the data is also required. Finally, in the case of deconvolution, thanks to discretization and quantization errors, the Fourier transform of the PSF is, in general, without zeros so that H is injective and uniqueness holds true.

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References

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