Abstracts of invited papers

Cracks, inclusions and other defects.

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We shall discuss inverse boundary value problems which model the detection of cracks, inclusions and cavities. We shall review recent results about uniqueness, stability and size evaluation.

Tauberian Theorems in Inverse Scattering

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Various Tauberian theorems, connecting the behaviour of the potential near the origin or at infinity to the behaviour of the phase-shifts for large or small momenta, are presented. These theorems are truly two-sided, and follow from Tauberian theorems in Fourier integral transforms and the Gel'fand-Levitan integral equation.

Blind Deconvolution as applied to Astronomical & Space Object Adaptive Optics Imaging

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In many cases for high resolution imaging through the atmosphere, no point spread function (PSF) is available or those that are only approximate the situation. This is particularly true in the case of Adaptive Optics (AO) imaging. The response of the system is not only dependant upon the instantaneous atmospheric parameters but also upon the source's brightness and extent. In addition Adaptive Optics systems are not perfect typically achieving Strehl ratios of 20-70%, perfect compensation would be 100%, so that deconvolution is necessary under many circumstances. However, the lack of a measured PSF or a poorly determined PSF necessitate the use of a "blind" deconvolution algorithm. As an ill-posed problem, a priori constraints are required to recover both object and PSF.

In this paper, we briefly outline why AO images need further reduction and discuss the reasons for the poorly determined PSF's. We also discuss how PSF estimates can be used with the observations to obtain higher Strehl images and also super-resolved images. The application of a blind-deconvolution algorithm is demonstrated with both simulated and real data and problems associated with its application are discussed. The use of different a priori constraints are discussed to aid in unique solutions for the algorithms convergence.

An Introduction to Recent Developments in Inverse Scattering Theory

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In recent years there has been considerable progress made in our understanding of inverse scattering problems for acoustic and electromagnetic waves. In this talk I will give a summary of some of these recent developments, concentrating on the simplest case of the scattering of a scalar time harmonic plane wave by an inhomogeneous medium of compact support. Topics discussed will include the uniqueness of the solution to the inverse problem, the spectral theory of the far field operator, and the regularized sampling method for numerically determining the support of the inhomogeneity. Such a discussion will require the introduction of Herglotz wave functions and their role in the study of interior transmission problems. Numerical examples will be given.

Inverting and super-resolving with wavelets

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In this talk,we start with a survey of the recent literature dedicated to the use of wavelet bases for solving inverse problems. We then recall that a critical issue in deriving regularized solutions is the question of resolution limits. A well-known challenge for some practical problems is to improve such resolution limits and in some instances to get what is called "super-resolution". Due to their good localization properties, wavelets are expected to be particularly suitable for this task. To support this idea, we present a generalization of the Gerchberg-Papoulis algorithm, which can be seen as a first step towards truly adaptive wavelet-based super-resolving algorithms.

Iterative Regularization of Nonlinear Inverse Probems with Applications to Parameter Identification

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The mathematical theory of regularization methods for nonlinear ill-posed problems is relatively new and by far not as complete as that for linear problems, which is mainly based on spectral theoretic methods. The first results were obtained for Tikhonov regularization, which has, however, severe drawbacks for nonlinear problems due to the fact that as opposed to the linear situation, the Tikhonov functional is in general non-convex and may hence have local minima. For nonlinear problems, iterative regularization methods seem to be an attractive alternative to Tikhonov regularization. However, the development of their convergence theory has only recently begun (cf. e.g. Engl-Hanke-Neubauer, Regularization of Inverse Problems, Kluwer 1996). In this talk, we present a new approach, based on invariance principles, to prove convergence rates for iterative regularization methods for solving nonlinear ill-posed problems, and illustrate this method for Landweber iteration and iteratively regularized Gauss-Newton methods. (Joint work with P.Deuflhard, Berlin and O.Scherzer, Linz). We then apply Landweber iteration to a parameter identification problem in a mathematical model for growth of polymers. (Joint work with M.Burger, Linz, and V.Capasso, Milano).

Some inverse problems in functional medical imaging

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The modern medical imaging techniques play a crucial role both in the diagnosis and therapy. Although the most conventional imaging techniques are associated with the concept of morphology, nowadays there are several relevant techniques which provide the information as a set of images representing the spatial distribution of some functional quantities. The most relevant methods of this type are Single Photon Emission Computed Tomography (SPECT), Positron Emission Tomography (PET) and functional Magnetic Resonance Imaging (fMRI).

From the physical point of view these "functional" imaging techniques may be very different but they have in common a chronic lack of information so that it is always difficult to extract the desired signals from noise.

From the mathematical point of view all these techniques require the solution of an integral transform which generates an ill-conditioned problem with a very large number of unknowns, when put in discrete form. The kernel of the integral transform is related to the specific technique; for instance, from a general point of view, SPECT and PET are described by the Radon transform whereas fMRI by the Fourier transform.

The most conventional approaches for solving these problems involve the straight application of discrete inverse transforms to the acquired data. However, the lack of information of functional imaging modalities requires the use of additional information and of appropriate mathematical techniques to embed it in the reconstruction of images.

In this work it will be shown how *a priori* information can be taken into account, for instance by modifying in an appropriate way the transform kernel. The reported examples were conceived keeping in mind the possibility of a clinical application of the techniques.

A numerical approach to the use of non measurable currents in inverse electromagnetic scattering of dielectric targets

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A major problem in dealing with inverse electromagnetic scattering of dielectric objects arises when one considers the field generated inside the object space by the so-called non radiating currents. Actually, such currents are invisible in the measurement space, but they do contribute in a not negligible manner to the field inside the object space, and they strongly depend on the scatterers under test, as well as on the interrogating source.

Although there is no-way to exactly reconstruct such currents from measurements, they can be taken into account when developing a numerical algorithm for inverse scattering. In particular, this communication presents a numerical approach that allows one to easily construct a basis for an approximation of the no-radiating currents. Such approximation actually provides one with non measurable currents, that do not produce any field at measurement points. The method starts by considering the integral equation of the scattering at measurement as well as at the object space. By performing a Singular Value Decomposition (SVD) of the operator relating the currents inside the scatterer with measurements, a basis is obtained that contains the measurable as well as the non measurable component of such currents. While the measurable currents are easily obtained by the equation at the measurement space, the non measurable components, as well as the dielectric features of the objects under test, are recovered in a further (second) step, by using a non linear procedure exploiting the equation at the object space.

Numerical simulations show that a notable improvement can be obtained with respect to the methods that disregard the non measurable currents.

Identifying a spatial body force in linear elastodynamics

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We present some results jointly obtained with Masahiro Yamamoto of University of Tokyo. Referring to the Linear Elasticity Theory, consider a sample of elastic and compressible material which occupies a bounded, connected, open set $\Omega \subset \mathbf{R}^n$, $n \geq 2$, for any time $t \in [0, T]$, where $T > 0$ is some fixed final time. We suppose that the material under consideration is subject to a body force $\mathbf{F} : [0, T] \times \Omega \to \mathbf{R}^n$ of the form $\mathbf{F}(t, x) := \varphi(t)\mathbf{f}(x)$. We recall that the evolution of the displacement vector field $\mathbf{u}$ is ruled by the well-known linear elastodynamic hyperbolic system. Assuming that $\mathbf{u}$ satisfies a given set of Cauchy-Dirichlet conditions, we would like to identify $\mathbf{f}$ by measuring the traction $\mathbf{g}$ exerted on a portion Γ of the boundary $\partial\Omega$ over the time interval $[0, T]$.

We first show that the linear mapping $\mathcal{G} : \mathbf{f} \to \mathbf{g}$ is Lipschitz continuous whenever T and the $(n-1)-$ dimensional Lebesgue measure of Γ are large enough. This theorem is proved by using an exact controllability inequality in the framework of Hilbert Uniqueness Method (HUM). An immediate consequence is that $\mathbf{g}$ uniquely determines $\mathbf{f}$. Moreover, still applying HUM techniques, we deduce a representation formula for $\mathbf{f}$ in terms of $\mathbf{g}$ and we provide a partial characterization of the range of the adjoint operator $\mathcal{G}^*$.

Mathematics and inverse problems: the two ends of the spectrum.

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I discuss two situations: the first one is a nice piece of math looking for applications in "inverse problems", the second one is exactly the opposite.

A new visualization technique in inverse scattering theory

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In this talk we present a numerical method for solving the inverse obstacle scattering problem for time-harmonic acoustic or electromagnetic waves. This method is extremely simple and fast and is able to reconstruct scatterers which consist of more than one component. On the other hand, it requires full information about the far field pattern. In the underlying analytical theory we give a new proof of uniqueness of the inverse scattering problem which is more explicit than the existing ones. We will show numerical reconstruction at the end of this talk.

Inverse Obstacle Scattering with Modified or Reduced Data

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We consider the basic inverse obstacle scattering problem for time-harmonic waves: recover the shape of the scatterer from the far field pattern for one incident direction and all observation directions. In the first part of the talk we will review some basic facts on a regularized Newton method for the approximate solution of this inverse problem. In the second part we will report on corresponding methods for the reconstruction from reduced or modified data such as limited-aperture observations, reconstructions from the amplitude of the far field only or reconstructions from backscattering data.

Linear and Nonlinear Scalar Inverse Scattering Comparison of Algorithms

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The solution of inverse scattering problems requires the choice of a diversity parameter of the incident field, which could either be frequency or the illumination angle (of a plane wave). In order to use the Ipswich-Data as experimental data we choose angular-diversity.

Linear inverse scattering relies on either the Born or the Kirchoff approximation for penetrable of perfectly scattering targets, respectively. An appropriate algorithm is in terms of filtered backpropagation. Applying it to Ipswich-Data of penetrable objects we obtain good results as long the Born approximation is satisfied, but the application to perfectly conducting data yields surprisingly good results, provided the data are not sparse.

Nonlinear inversion is formulated for perfectly conducting targets as an iterative scheme being linearized in each iteration step; in addition, the Fréchet derivative is computed within a linearizing approximation. Nevertheless, the resulting hermitian matrix is ill-conditioned and has to be regularized, where we determine the regularization parameter via the trace of the matrix and the error norm. We observe good convergence for the Ipswich-Data, even for sparse data. But: The increased computation time for nonlinear inversion does not justify the somewhat better resolution as compared to linear inversion, except for the sparse data case.

Eddy current nondestructive evaluation using SQUIDs: from theory to laboratory-controlled experiments.

A collective research contribution by
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The interest in high temperature SQUID (Superconducting QUantum Interference Device)-based gradiometers for eddy current nondestructive evaluation, for example, of planar stratified structures such as those commonly encountered in the aircraft industry, is originated from the high sensitivity of SQUIDs to the magnetic flux at very low frequencies (A. Ruosi et al., "Eddy-current nondestructive measurements using SQUIDs: preliminary results," in Electromagnetic Nondestructive Evaluation (II), R. Albanese et al. Eds., 215-224, IOS Press, Amsterdam, 1998).

In this contribution, we intend to sketch how rather complex theoretical, numerical and measurement machineries are to be combined in order to get reasonable synthetic and experimental data which enables us first to reach a good understanding of the complex interaction of diffusive wavefields (here, generated by a non-magnetic current source) with a strongly conductive, linear isotropic non-magnetic slab affected by one or more defects, and second to lay the ground for the retrieval of pertinent features of the defects. The expose itself will develop as follows.

After a short tour of the context of the electromagnetic non-destructive evaluation, we will show how a rigorous domain-integral vector wavefield formulation making use of appropriate dyadic Green's functions (as done earlier for a circular cylindrical structure by V. Monebhurrun et al., "Evaluation of a 3-D bounded defect in the wall of a metal tube at eddy current frequencies: the direct problem," J. Electromagn. Waves Applic. 12 (1998) 315-347) provides us with excellent tools to model the anomalous fields that are induced by the presence of a defect in a slab under interrogation.

Different levels of approximations such as the quasi-static field hypothesis and the localized nonlinear approximation will also be introduced and justified with respect to the full solution in particular since the high computational cost of this full solution does not make it a likely candidate for inversion.

Then we will focus onto the measurement modalities and illustrate to some detail how laboratory-controlled experiments can be conducted by means of a carefully-designed SQUID-based probe displaced above the damaged slab. The reliability and the accuracy of such experiments will be discussed, and the acquired data will be confronted to those acquired by more traditional probes, and to simulation results derived from the above integral approach and from a finite-element code available elsewhere. In so doing we will in particular attempt to characterize shape and limitations of the experimental data since inversion algorithms must closely take cognizance of them.

To conclude, we will briefly analyze, and illustrate at least from preliminary results, how (and far from a mere detection) the contrast map of a void defect can be retrieved in a iterative fashion by variants of the modified gradient method with using the fact that this defect and its immediate embedding can be represented by a binary function valued at 1 inside the defect and 0 outside (refer to the analysis of V. Monebhurrun et al., "Inversion of eddy current data for Nondestructive Evaluation of steam generator tubes," Inverse Problems 14 (1998) 707-724).

Integrodifferential identification problems at resonance

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We consider the problem of identifying the memory kernel $h : [0, T] \to \mathbf{R}$ in the integrodifferential equation

$$A_1 u'(t) - A_2 u(t) - \int_0^t h(t-s) A_3 u(s)\, ds = f(t) \qquad t \in [0, T]$$

subject to the *initial condition* $u(0) = u_0$ and the *additional information* $\Phi[A_1 u(t)] = g(t)$ for any $t \in [0, T]$.

Operators A_j $(j = 1, 2, 3)$ are assumed to be linear and closed in suitable *Hilbert* spaces and to satisfy the following basic assumptions: *i)* the A_j's are operators associated with sesquilinear Hermitian forms a_j $(j = 1, 2, 3)$, a_2 being *coercive*; *ii)* the kernel of A_1 is *not* empty; *iii)* Φ is a linear continuous functional.

Under the condition $\Phi[A_3 u_0] \neq 0$ and some further additional assumptions concerning the Hilbert spaces under considerations and the operators A_j we can prove that our identification problem admits a unique solution continuously depending on the data (f, u_0, g).

Some applications are given in the case where A_1 and A_2, A_3 are second– and fourth–order linear differential operators with variable coefficients.

A particular attention is paid to the case when $A_1 = \lambda - P_1(A)$, $A_2 = P_2(A)$, $A_3 = P_3(A)$. Here, operator A is assumed to be *positive* and *self-adjoint* with a compact inverse, λ is an eigenvalue of $P_1(A)$ and P_j $(j = 1, 2, 3)$ are three *given* polynomials with *constant* coefficients.

We stress that, in this case, the usual technique consisting in "dividing" $P_1(A)$ by $\lambda - P_0(A)$ *cannot* (in general) be followed since $\lambda - P_0(A)$ is *not* invertible.

A Fast Method for the Inverse Scattering Problem

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The induced sources in the Helmholtz equation are the solution of a linear problem. An efficient solution algorithm is based on the approximate inverse where a reconstruction kernel is precomputed.

Using the object equation the scatterer can be computed from the induced sources. Examples for the 2D case are presented.

Old printed texts, character recognition and inverse problems in image processing

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Reliable methods for converting printed text into ASCII- or even LATEX-files are well established for modern text fonts. These methods however fail when applied to old fonts such as "Frakturschrift", which was frequently used until the middle of 19th century.

This problem can be treated by a variety of mathematical tools from image processing such as non-linear diffusion equations and wavelet techniques. These methods have to be adapted to the special structure of the underlying problem. Results from real data demonstrate the efficiency of the reulsting algorithm.

Inverse boundary value and inverse scattering problems in two dimensions

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This talk will describe the uniqueness and reconstruction proof in the two dimensional inverse conductivity problem. An overview of related results on stability (joint with L.Liu), linear and nonlinear Schroedinger operators (joint with V.Isakov) will also be given.

Newton-like Methods, Bounded Variation Regularization and Variational Inequalities for Ill-Posed Problems

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This talk will focus on some aspects of recent results by the author and his collaborators on three topics: Newton-like methods for smooth and non-smooth ill-posed operator equations using adaptive constructs of outer inverses; least squares and bounded variation regularization with nondifferentiable functionals; and ill-posed variational inequalities.

Inversion formulas in vector tomography

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In vector tomography one reconstructs a vector field from line integrals. Applications occur e.g. in flow, Schlieren, and photoelastic tomography. In the talk we give a unified derivation of inversion formulas for the vectorial ray transform, the Radon probe transform, and the Radon normal transform obtained by Braun and Hauck, Sharafutdinov, Juhlin, Prince, Sparr and Strahlen, and others. We use only elementary facts about Fourier transforms and the well known Fourier slice Theorems for the scalar Radon and ray transform of classical tomography.

On recovering singularities from the back-scattering data

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This is a joint work with Petri Ola from Oulu and Valeri Serov from Moscow. We consider the inverse back-scattering problem for the Schrödinger equation. It is a famous open problem whether the back-scattering amplitude uniquely determines the scattering potential. Partial results were obtained in late 60's by Protter and in late 80's by Ralston and Eskin. Protter proved the uniqueness for small potentials and Ralston and Eskin the generic uniqueness.

In this talk we are interested in a more modest problem of determining the singularities of the potential from the back-scattering data. Our result is in two dimensions. It says that the difference of the original potential and the Born approximation (inverse Fourier transform of the back-scattering amplitude) is in a smoother Sobolev class than the original potential. In other words the Born approximation, that is obtained directly and linearly from the data, carries the same singularities as the original potential. Earlier result of this direction was obtained by A. Greenleaf and G. Uhlmann who showed in the time dependent case that the conormal singularities over smooth surfaces can be recovered from the singularities of the scattering data. Our method is based on a new characterization of the Sobolew space H^1 by P. Hajlasz and on a pointwise maximal inequality of H. Triebel.

Approaches to Image reconstruction in Electrical Impedance Tomography

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Electrical Impedance Tomography is one of the practical applications and implementations of mathematical results relating to the inverse conductivity problem. The technique has potential applications in medicine and a number of industrial uses have also been identified. From a mathematical point of view the image reconstruction problem is easily and simply stated but it is well known to represent a highly ill-posed non-linear inverse problem. It has attracted much attention from mathematicians and engineers and work has been published reflecting a wide range of mathematical sophistication and practical orientation. Many image reconstruction algoritms have been proposed which also differ widely in their styles and ambitions. In this talk I will discuss a number of aspects of the image reconstruction process in Electrical Impedance Tomography and describe some of the range of approaches that I have been exploring with different colleagues over the last year.

Quadratic approaches to inverse scattering

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The problem of reconstructing the dielectric contrast profile of a two dimensional dielectric cylinder from scattered field measurements is, as well known, a nonlinear ill posed problem. A method to deal with it consists in expanding the contrast by a finite set of basis functions and looking for a finite number of unknown coefficients by minimising an error function. Generally, this error function presents several local minima, traps for the inversion procedure.

A widely used simplification is obtained by linearising the operator mapping the contrast onto the scattered field. However, a linear inversion does not allow to reconstruct spatial variations related to the higher order functions of the Singular Value Decomposition (SVD) of the linearised operator since the singular functions corresponding to the lower singular values have a negligible image. This generally provides limitations on the reconstruction of rapidly varying profiles.

Recently, a new approach based on a quadratic approximation has been considered [1] and its capability of retrieving a wider class of contrast profiles with respect of the linear inversion methods has been shown.

The quadratic approach provides an error function of the fourth order which makes it possible to get an insight on the local minima problem. In fact it has been shown [2] that by increasing the number of independent measurements with respect to the number of unknowns the error function has only one (global) minimum.

We extended the quadratic inversion approach to the iterated method, where at each step only the difference with respect to the contrast function found at the previous iteration is searched for. This procedure can enlarge the class of reconstructable profiles.

[1] R. Pierri and A. Brancaccio, "Imaging of a rotationally symmetric cylinder by a quadratic approach", J. Opt. Soc. Amer., A 14, 2777-2785 (1997).

[2] R. Pierri and A. Tamburrino, "On the local minima problem in conductivity imaging via a quadratic approach", Inverse Problems, 13, 1-22 (1997).

The use of a dual method of mathematical programming for data inversion in linear ill-posed problems.

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Work will be presented here which attempts to add in some information lost in the data of an ill-posed problem by making use of the a-priori constraints of positivity and known moments. This is achieved by the dual method of quadratic programming using some spectacular new mathematical results in this field due to Miccelli and to Borwein and Lewis.

Computations in aero-optics image recovery

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Real-time adaptive-optics and off-line restoration are means for enhancing the resolution of optical images beyond the limits imposed by the turbulent atmosphere. Applied as an image of an object is formed, adaptive-optics (AO) techniques compensate for degradations added along the path of the light from the object being imaged. One approach for performance modeling of closed-loop adaptive-optics systems involves calculating very large covariance matrices whose components can be represented by Hankel transform based integrals. We develop approximate matrix factorizations of discretizations of these integrals, enabling fast schemes for both computing and applying the covariance matrices, using fast cosine transforms and separable fast multipole methods.

Our image restoration work involves phase diversity-based blind deconvolution. Phase diversity (PD) is a technique for obtaining estimates of both the object and the phase, by exploiting the simultaneous collection of two (or more) short-exposure optical images, one of which has been formed by further blurring the conventional image in some known fashion. We provide a fast computational algorithm based upon a regularized variant of the Gauss-Newton optimization method, leading to robust and numerically efficient restoration of optical images.

The speaker's research is supported by the AFOSR and the NSF. Parts of the lecture represent work with co-authors T. Chan, B. Ellerbroek, P. Pauca, N. Pitsianis, X. Sun and C. Vogel.

An Inverse Spectral Problem arising in Helioseismology

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We consider the problem of determining a radially-symmetric potential in the three dimensional Schrödinger equation from data consisting of eigenvalues associated with spectral sequences from two different angular momentum quantum numbers. In recent years enormous numbers of quite accurate eigenvalues of the sun have been compiled. Our problem represents one of the simplest that model the determination of certain physical parameters in the standard model of the sun from eigenvalue data. This leads to a highly singular eigenvalue problem for which there are no known uniqueness to the most reasonable conjectures. We are able to give strong evidence for uniqueness in many cases and discuss a computational solution method together with some supporting analysis.

Inversion Equations and Solutions of integrable nlpde

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Inversion equations used to solve integrable nlpde are always of the Marchenko form, i.e explicitly use information on the conditions at infinity. Here it is first shown that other inversion equations, e.g. of the Gelfand Levitan form, also yield solutions. A comparative analysis is done, in the case of KdV, of the two approaches.

Inverse Scattering of Generalized Schrödinger Equations and First Order Systems on the Line

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We give a review of exact inverse scattering techniques for the generalized Schrödinger equation

$$\psi''(k,x) + k^2 H(x)\psi(k,x) = Q(x)\psi(k,x), \qquad x \in \mathbf{R}, \tag{1}$$

where $H(x)$ is strictly positive and has finitely many jump discontinuities, the modified Schrödinger equation

$$\psi''(k,x) + k^2\psi(k,x) = [ikP(x) + Q(x)]\,\psi(k,x), \qquad x \in \mathbf{R}, \tag{2}$$

where $P(x)$ and $Q(x)$ are real potentials, and the so-called canonical differential system

$$-iJ\psi'(x,\lambda) - V(x)\psi(x,\lambda) = \lambda\psi(x,\lambda), \qquad x \in \mathbf{R}, \tag{3}$$

where

$$J=\begin{bmatrix} I_n & 0 \\ 0 & I_n \end{bmatrix}, \qquad V(x)=\begin{bmatrix} 0 & k(x) \\ k(x)^\dagger & 0 \end{bmatrix}.$$

Having defined the scattering solutions and the scattering matrix, the problem is to reconstruct the potential(s) from one of the reflection coefficients and certain bound state information uniquely and to give necessary conditions and sufficient conditions on the scattering data to recover potential(s) with certain integrability conditions. Typically, the inverse scattering problem is first written as a vector Riemann-Hilbert problem on the line, and then converted into a Marchenko integral equation. Conditions are given for the unique solvability of this equation and system theoretic methods are applied to find the explicit solution when the given reflection coefficient is rational.

As an example, for Eq. (3) and $V(x)$ having entries in $L^1(\mathbf{R})$, we define the Jost solutions $F_{l,r}(x,\lambda)$ by requiring that

$$F_l(x,\lambda)=e^{i\lambda Jx}[I_{2n}+o(1)], \qquad x\to+\infty;$$

$$F_r(x,\lambda)=e^{i\lambda Jx}[I_{2n}+o(1)], \qquad x\to-\infty.$$

These solutions satisfy

$$F_l(x,\lambda)=e^{i\lambda Jx}A_l(\lambda)+o(1), \qquad x\to-\infty;$$

$$F_r(x,\lambda)=e^{i\lambda Jx}A_r(\lambda)+o(1), \qquad x\to+\infty,$$

where $A_l(\lambda)^{-1}=A_r(\lambda)=JA_l(\lambda)^\dagger J$ can be partitioned into $n\times n$ blocks as follows:

$$A_l(\lambda)=\begin{bmatrix} T_l(\lambda)^{-1} & -T_l(\lambda)^{-1}R(\lambda) \\ L(\lambda)T_l(\lambda)^{-1} & [T_r(\lambda)^\dagger]^{-1} \end{bmatrix}.$$

The problem is to reconstruct the potential $k(x)$ from the reflection coefficient $R(\lambda)$ using the Marchenko equations

$$B_{l2}(x,\alpha)=-\hat{R}(\alpha+2x)+\int_0^\infty d\beta\int_0^\infty d\gamma\, B_{l2}(x,\gamma)\hat{R}(\beta+\gamma+2x)^\dagger\hat{R}(\alpha+\beta+2x), \tag{4}$$

$$B_{r3}(x,\alpha)=-\hat{L}(\alpha-2x)+\int_0^\infty d\beta\int_0^\infty d\gamma\, B_{r3}(x,\gamma)\hat{L}(\beta+\gamma-2x)^\dagger\hat{L}(\alpha+\beta-2x), \tag{5}$$

where $\alpha \geq 0$ and $k(x) = 2i\, B_{l2}(x, 0^+) = -2i\, B_{r3}(x, 0^+)^\dagger$.

When $R(\lambda)$ is a rational matrix function without real zeros that is a strict contraction for $\lambda \in \mathbf{R}$ and vanishes as $\lambda \to \pm\infty$, Eq. (4) has a separated kernel if $x > 0$ and Eq. (5) has a separated kernel if $x < 0$. The inversion algorithm then is to solve Eq. (4) for $x > 0$ yielding $k(x)$ for $x > 0$, to complete the scattering matrix

$$\mathbf{S}(\lambda) = \begin{bmatrix} T_l(\lambda) & R(\lambda) \\ L(\lambda) & T_r(\lambda) \end{bmatrix}$$

to a unitary matrix of order $2n$ yielding $L(\lambda)$, and to solve Eq. (5) for $x < 0$ resulting in $k(x)$ for $x < 0$.

The usual Schrödinger equation and Eq. (2) can be solved explicitly for rational reflection coefficient using similar techniques. Equation (3) involves large k asymptotics of the scattering coefficients that are almost periodic polynomials. The inversion of this equation will be discussed in some detail.

Joint work with Tuncay Aktosun of North Dakota State University and Martin Klaus of Virginia Tech.

Numerical Methods for Determination of Physical Parameters and Potentials

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In this talk, we will present some of our recent theoretical and numerical results in the determination of **(a)** physical parameters in elliptic and parabolic systems and **(b)** potentials in a conservative or nonconservative system from the observed spectra and normal modes. Basic approach will be a combination of the augmented Lagrangian method and the equation error method with a H^1-norm or BV-norm regularization. The nonlinear minimization system is discretized by finite element methods. The existence of minimizers to both the continuous and discretized optimization problems and the convergence of the discretization are proved. Numerical experiments will be given to demonstrate the efficiency of the proposed methods for identifying highly discontinuous and oscillating parameters and potentials.

Finally, we will discuss one of the most popular topics in numerically solving inverse problems – the choice of optimal regularization parameters. Blind choice of these parameters may result in tremendously different accuracy of numerical solutions. We propose an efficient method for choosing reasonable regularization parameters.

Contributed papers

Identification of thin inhomogeneities by electrostatic measurements.

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We formally derive an asymptotic formula for the electrostatic voltage potential in the presence of a finite number of thin inhomogeneities in a conducting body. We use this formula to design an effective identification procedure.

Design and implementation issues of image processing algorithms in a parallel and distributed memory environment

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Among the fields of interest in Inverse Problems, Image Processing applications play a leading role, due to the strong impact that they have in real world. Indeed, in recent years, there has been a tremendous increase in the demand for digital imagery. Applications include telecommunication, medical diagnosis, astronomy, scientific visualization.

The increasing availability of advanced architecture computers is having a significant effort in all spheres of scientific computation, concerning Image

Processing applications this results either in high resolution images (up to 6.000 $\times$ 6.000 pixels) and sequences of images (20/30 frames/sec.) or in real-time processing.

Although high performance computing environments have become available, these machines have non achieved widespread use, especially outside of accademic research environments. This slowness seems to be caused by the lack of efficient and portable mathematical software.

In this talk, we survey some of basic ideas about the analysis, design and implementation of algorithms for Image Processing in a parallel environment. In particular, we focus our attention on MIMD distributed memory machine, that are presently considered the most suitable architectures for solving large-scale problems.

Two strategies for introducing parallelism into algorithms will be considered, *the parallelization of the building blocks* and *the domain decomposition.* Both strategies are generally coupled with suitable balancing techniques, so that each processor has the same amount of work and communication strategies in order to reduces the overhead time. The key insight of the parallel approach is the choice of sequential strategies which seem to be the most promising for the advanced architecture computers and the re-use of standard software for the parallelization of the fundamental building blocks.

We try to explain these concepts by considering the *restoration* of images from degraded observations, whose mathematical model is an integral equation of first kind. We consider two approaches, one based on the use of the Fast Wavelet Transform for the numerical solution of the discrete model, and the other based on the use of Total Variation as regularization functional.

Regularization methods for the solution of inverse problems in medical imaging.

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Some problems in the area of medical imaging, such as the image reconstruction in Computed Tomography (SPECT and TAC) or the functional Magnetic Resonance Imaging (fMRI), are modelled by Fredholm integral equations of the first kind with general kernel and additive white noise. The discretization of the integral equations with general kernel leads to larg linearize systems with ill conditioned coefficient matrix with no particolar structure. Hence, regularization methods are necessary to solve the systems. We analyse some regularization methods and the choice of the relative regularization parameters when applied to data coming from SPECT, TAC and FMRI. A parallel implementation on CRAY T3E is proposed in order to reduce the high computational complexity.

Poster Session

Paola Bonetto
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Fast inversion algorithms for SPECT data based on a simple model of the projection matrix

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An identification problem related to a parabolic integrodifferential equation with non commuting spatial operators.

Richard Kowar
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On the Landweber iteration for the solution of a parameter identification problem in a hyperbolic partial differential equation of the second order.

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Determination of boundary defects by electrostatic measurements

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Optimal size estimates for the inverse conductivity problem with one measurement.